A Solution of F subproblem

$\mathbf{F}$ subproblem is written as

$$\begin{aligned} &\min_{\mathbf{F}} \|\mathbf{F}-\mathbf{P}\|_F^2 + \alpha \left\|\mathbf{A}\odot \mathbf{F}\mathbf{F}^\top\right\|_1 \\ &\text{s.t. } \mathbf{F}\mathbf{1}_q = \mathbf{1}_m, \mathbf{0}_{m\times q} \le \mathbf{F} \le \mathbf{Y}, \end{aligned} \tag{1}$$

where $\mathbf{P} = \mathbf{K}\mathbf{H} + \mathbf{1}_m \mathbf{b}^\top \in \mathbb{R}^{m\times q}$ is the output matrix of the model. We first reformulate Eq. (1) column-wisely:

$$\begin{aligned} &\min_{\mathbf{F}} \alpha \sum_{i=1}^{q} \mathbf{F}_{\cdot i}^\top \mathbf{A}\mathbf{F}_{\cdot i} + \sum_{i=1}^{q} \mathbf{F}_{\cdot i}^\top \mathbf{F}_{\cdot i} - 2\sum_{i=1}^{q} \mathbf{P}_{\cdot i}^\top \mathbf{F}_{\cdot i} \\ &\text{s.t. } \mathbf{F}\mathbf{1}_q = \mathbf{1}_m, \mathbf{0}_{m\times q} \le \mathbf{F} \le \mathbf{Y}, \end{aligned} \tag{2}$$

where $\mathbf{F}_{\cdot i}$ and $\mathbf{P}_{\cdot i}$ are the i-th column of $\mathbf{F}$ and $\mathbf{P}$ respectively. Denote $\mathbf{f} = vec(\mathbf{F}) \in \mathbb{R}^{mq}$, $\mathbf{p} = vec(\mathbf{P}) \in \mathbb{R}^{mq}$, $\mathbf{y} = vec(\mathbf{Y}) \in \mathbb{R}^{mq}$, where $vec(\cdot)$ is the vectorization operator. Eq. (2) can be further formulated as

$$\begin{aligned} &\min_{\mathbf{f}} \frac{1}{2}\mathbf{f}^\top \left(2\mathbf{\Lambda} + \frac{2}{\alpha}\mathbf{I}_{mq\times mq}\right)\mathbf{f} - \frac{2}{\alpha}\mathbf{p}^\top \mathbf{f} \\ &\text{s.t. } \sum_{\substack{j=1,\\ j\%m=i}}^{mq} \mathbf{f}_j = 1 (\forall\, 0 \le i \le m-1), \mathbf{0}_{mq} \le \mathbf{f} \le \mathbf{y}, \end{aligned} \tag{3}$$

where $\%$ denotes the modulo operator, $\mathbf{f}_j$ is the j-th element of vector $\mathbf{f}$ and $\mathbf{\Lambda} \in \mathbb{R}^{mq\times mq}$ is defined as

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{m\times m} & \cdots & \mathbf{0}_{m\times m} \\ \mathbf{0}_{m\times m} & \mathbf{A} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0}_{m\times m} \\ \mathbf{0}_{m\times m} & \cdots & \mathbf{0}_{m\times m} & \mathbf{A} \end{bmatrix}. \tag{4}$$

Eq. (3) is a standard quadratic programming (QP) problem, which can be solved by any QP tools.

Improve the Scalability

Eq. (3) solves a QP problem with the computational complexity of $\mathcal{O}(m^3q^3)$ and the storage complexity of $\mathcal{O}(m^2q^2)$. To reduce the computational complexity and the storage complexity, we can approximately solve the original QP problem row-wisely, i.e., update the label confidence vector $\widetilde{\mathbf{f}}_j$ by fixing other variables:

$$\begin{aligned} &\min_{\widetilde{\mathbf{f}}_j} \frac{1}{2}\widetilde{\mathbf{f}}_j^\top (2\mathbf{D}_{jj} + \frac{2}{\alpha})\mathbf{I}_{q\times q}\widetilde{\mathbf{f}}_j + (\sum_{i=1, i\neq j}^{m} \mathbf{D}_{ij}\widetilde{\mathbf{f}}_i^\top - \frac{2}{\alpha}\mathbf{p}_j^\top)\widetilde{\mathbf{f}}_j \\ &\text{s.t. } \widetilde{\mathbf{f}}_j \mathbf{1}_q = 1, \mathbf{0}_q \le \widetilde{\mathbf{f}}_j \le \mathbf{y}_j. \end{aligned} \tag{5}$$

By this way, the original QP problem is transformed into a series of small QP problems, and the computational complexity of the QP step is reduced from $\mathcal{O}(m^3q^3)$ to $\mathcal{O}(mq^3)$, and the storage complexity is reduced from $\mathcal{O}(m^2q^2)$ to $\mathcal{O}(q^2)$.

Computational Complexity Comparison among the Regression based PLL Methods

Table S1 compares the computational complexity between the regression based PLL methods, i.e., AGGD (TPAMI 2022), PL-CLA (JCST 2021), SDIM (IJCAI 2019), DPCLS, and DPCLS-S (Scalable DPCLS). DPCLS solves a QP problem with the computational complexity of $\mathcal{O}(m^3q^3)$, which is the same as many SOTA PLL methods like AGGD, SDIM, and PL-CLA. DPCLS-S transforms the original QP problem into a series of smaller QP problems, and the computational complexity of the QP step is reduced from $\mathcal{O}(m^3q^3)$ to $\mathcal{O}(mq^3)$, and the storage complexity is reduced from $\mathcal{O}(m^2q^2)$ to $\mathcal{O}(q^2)$. This approximation solution only slightly decreases the accuracy as shown in Table S2, but largely improves the scalability.

Table S1: Computational complexity comparison between the linear regression based PLL methods.

	AGGD	PL-CLA	SDIM	DPCLS	DPCLS-S
Computational complexity	$\mathcal{O}(m^3+mk^3+m^3q^3)$	$\mathcal{O}(m^3+m^3q^3)$	$\mathcal{O}(m^3+m^3q^3)$	$\mathcal{O}(2m^3+m^2+m^3q^3)$	$\mathcal{O}(2m^3+m^2+mq^3)$

Table S2: Comparison between DPCLS and DPCLS-S, where DPCLS-S indicates scalable DPCLS.

	Glass	Ecoli	Steel	Yeast	Optdigits	Usps
DPCLS	**.560±.051**	**.833±.014**	**.638±.022**	**.507±.026**	**.984±.002**	**.968±.004**
DPCLS-S	.544±.049	.821±.023	.633±.022	.496±.018	.982±.001	.966±.004

B Proof of Theorem 1 and Theorem 2

B1. Proof of Theorem 1

Definition 1. *Denote $\mathcal{H}$ be a family of functions that map $\mathcal{X}$ to $[0,1]$ and $S=\{x_1,x_2,...,x_m\}$ is a set of fixed samples. The empirical Rademacher complexity of $\mathcal{H}$ to set S is defined as*

$$\hat{\mathcal{R}}_S(\mathcal{H})=\frac{1}{m}\mathbb{E}\left[\sup_{h\in\mathcal{H}}\sum_{i=1}^{m}\sigma_i h(x_i)\right], \tag{6}$$

where $(\sigma_1,\sigma_2,...,\sigma_m)$ are Rademacher variables, and each of them is an independent uniform random variable taking value in $\{-1,+1\}$.

The square loss function of DPCLS is $\|\mathbf{F}-\mathbf{XW}\|_F^2$. $\mathbf{F}$ can be divided into the sum of the ground-truth label matrix $\mathbf{F_G}\in\mathbb{R}^{m\times q}$ and false-positive label matrix $\mathbf{N}\in\mathbb{R}^{m\times q}$. So we can rewrite the square loss function as $\|\mathbf{F_G}+\mathbf{N}-\mathbf{XW}\|_F^2$. Based on **Definition 1**, let $\mathcal{H}=\mathbf{W}\times\mathbf{N}$ be the family of functions of DPCLS, i.e., $(\mathbf{W},\mathbf{N})\in\mathcal{H}$. ℓ is the square loss function of DPCLS, the Rademacher complexity with respective to $\mathcal{H}$ and ℓ can be expressed as

$$\hat{\mathcal{R}}_S(\ell\circ\mathcal{H})=\frac{1}{m}\mathbb{E}\left[\sup_{h\in\mathcal{H}}\sum_{i=1}^{m}\sigma_i\ell(h(\mathbf{x}_i),\mathbf{F_G}_i)\right]. \tag{7}$$

Note that the square loss is $2q$-Lipschitz. According to [1], Eq. (7) is upper bounded by

$$\hat{\mathcal{R}}_S(\ell\circ\mathcal{H})\le\frac{2\sqrt{2}q}{m}\mathbb{E}\left[\sup_{h\in\mathcal{H}}\sum_{i=1}^{m}\sum_{j=1}^{q}\sigma_{ij}(x_i\mathbf{W}_{\cdot j}-\mathbf{N}_{ij})\right], \tag{8}$$

where σ_{ij} is the Rademacher variable which takes value in $\{-1,1\}$. $\mathbf{W}_{\cdot j}$ is j-th column of classifier $\mathbf{W}$. Denote $\hat{\mathbf{X}}=[\hat{\mathbf{X}}_1;\hat{\mathbf{X}}_2;...;\hat{\mathbf{X}}_q]\in\mathbb{R}^{q\times d}$, $\hat{\mathbf{X}}_q=\sum_{i=1}^{n}\sigma_{iq}x_i$. Without loss of generality, we assume the complexity of classifier $\mathbf{W}$ and the sparsity of $\mathbf{N}$ are upper bounded by ϵ_1 and ϵ_2 respectively, i.e., $\|\mathbf{W}\|_F\le\epsilon_1$ and $\|\mathbf{N}\|_1\le\epsilon_2$. The right side of Eq. (8) can be relaxed as:

$$\begin{aligned}\hat{\mathcal{R}}_S(\ell\circ\mathcal{H})&\le\frac{2\sqrt{2}q}{m}\mathbb{E}\left[\sup_{h\in\mathcal{H}}\langle\mathbf{W}^{\mathsf{T}},\hat{\mathbf{X}}\rangle+\|\mathbf{N}\|_1\right]\\&\le\frac{2\sqrt{2}q}{m}\mathbb{E}\left[\sup_{h\in\mathcal{H}}\|\mathbf{W}\|_F\left\|\hat{\mathbf{X}}\right\|_F+\|\mathbf{N}\|_1\right]\\&\le\frac{2\sqrt{2}q}{m}\mathbb{E}\left[\sup_{h\in\mathcal{H}}\epsilon_1\left\|\hat{\mathbf{X}}\right\|_F+\epsilon_2\right].\end{aligned} \tag{9}$$

We assume each sample is normalized, i.e., $\|x_i\|_2\le 1$, it is easy to prove that

$$\mathbb{E}_\sigma\left\|\hat{\mathbf{X}}\right\|_F^2=\mathbb{E}_\sigma\left[\sum_{j=1}^{q}\left\|\hat{\mathbf{X}}_j\right\|_2^2\right]=\mathbb{E}_\sigma\left[\sum_{j=1}^{q}\left\|\sum_{i=1}^{m}\sigma_{ij}\mathbf{x}_i\right\|_2^2\right]\le mq. \tag{10}$$

According to Eq. (9) and Eq. (10) we have **Theorem 1**

$$\hat{\mathcal{R}}_S(\ell\circ\mathcal{H})\le\frac{2\sqrt{2}q(\sqrt{mq}\epsilon_1+\epsilon_2)}{m}. \tag{11}$$

B2. Proof of Theorem 2 Inequality 1

Denote $\mathbf{F} \in \{0,1\}^{m\times q}$ and $\mathbf{D} \in \{0,1\}^{m\times m}$ the partial label matrix and the to be optimized semantic dissimilarity matrix. Let $\mathbf{F_G}$ and $\hat{\mathbf{D}}$ be the ground-truth label matrix and the ground-truth dissimilarity matrix. Denote $\mathbf{\Delta_F} = \mathbf{F_G} - \mathbf{F}$. We aim to minimize the adversarial prior between the semantic dissimilarity matrix and similarity matrix, i.e., $\left\|\mathbf{D} \odot \mathbf{F}\mathbf{F}^\mathsf{T}\right\|_1$. The following inequality holds

$$\langle (\mathbf{F}+\mathbf{\Delta_F})(\mathbf{F}+\mathbf{\Delta_F})^\mathsf{T}, \hat{\mathbf{D}}\rangle \leq \langle \mathbf{F}\mathbf{F}^\mathsf{T}, \mathbf{D}\rangle. \tag{12}$$

Expand Eq. (12), we have

$$\begin{aligned}\langle \mathbf{\Delta_F}\mathbf{\Delta_F}^\mathsf{T}, \hat{\mathbf{D}}\rangle &\leq \langle \mathbf{F}\mathbf{F}^\mathsf{T}, \mathbf{D}-\hat{\mathbf{D}}\rangle - \langle \mathbf{F}\mathbf{\Delta_F}^\mathsf{T}, \hat{\mathbf{D}}\rangle - \langle \mathbf{\Delta_F}\mathbf{F}^\mathsf{T}, \hat{\mathbf{D}}\rangle \\ &= \langle \mathbf{F}\mathbf{F}^\mathsf{T}, \mathbf{D}-\hat{\mathbf{D}}\rangle + \langle \mathbf{F}\mathbf{\Delta_F}^\mathsf{T}, -\hat{\mathbf{D}}\rangle + \langle \mathbf{\Delta_F}\mathbf{F}^\mathsf{T}, -\hat{\mathbf{D}}\rangle \\ &\leq \|\mathbf{F}\|_F^2 \left\|\mathbf{D}-\hat{\mathbf{D}}\right\|_F + 2\|\mathbf{F}\|_F \left\|\hat{\mathbf{D}}\right\|_F \|\mathbf{\Delta_F}\|_F .\end{aligned} \tag{13}$$

Due to the characteristics of PLL, at least one false-positive label exists in each PLL data set, i.e., $\|\mathbf{\Delta_F}\|_F \geq 1$. In this way, we have

$$\langle \mathbf{\Delta_F}\mathbf{\Delta_F}^\mathsf{T}, \hat{\mathbf{D}}\rangle \leq \|\mathbf{F}\|_F^2 \left\|\mathbf{D}-\hat{\mathbf{D}}\right\|_F \|\mathbf{\Delta_F}\|_F + 2\|\mathbf{F}\|_F \left\|\hat{\mathbf{D}}\right\|_F \|\mathbf{\Delta_F}\|_F . \tag{14}$$

Assume the smallest eigenvalue of $\hat{\mathbf{D}}$ is $\lambda_{\hat{\mathbf{D}}}$ and $\lambda_{\hat{\mathbf{D}}} \geq 0$, we have $\langle \mathbf{\Delta_F}\mathbf{\Delta_F}^\mathsf{T}, \hat{\mathbf{D}}\rangle \geq \lambda_{\hat{\mathbf{D}}} \|\mathbf{\Delta_F}\|_F^2$. Moreover, $\|\mathbf{F}\|_F^2$ is upper bounded by m (the number of samples) and q (the number of classes), i.e., $\|\mathbf{F}\|_F^2 \leq mq$. Eq. (14) can be further relaxed as

$$\lambda_{\hat{\mathbf{D}}} \|\mathbf{\Delta_F}\|_F^2 \leq mq \left\|\mathbf{D}-\hat{\mathbf{D}}\right\|_F \|\mathbf{\Delta_F}\|_F + 2\sqrt{mq} \left\|\hat{\mathbf{D}}\right\|_F \|\mathbf{\Delta_F}\|_F . \tag{15}$$

Let $\left\|\bar{\mathbf{\Delta}}_\mathbf{F}\right\|_F$ be the average distance of each sample between $\mathbf{F_G}$ and $\mathbf{F}$ (i.e., $\left\|\bar{\mathbf{\Delta}}_\mathbf{F}\right\|_F = \frac{1}{m}\|\mathbf{F_G}-\mathbf{F}\|_F$). Dividing m on both sides of Eq. (15), we finally have

$$\left\|\bar{\mathbf{\Delta}}_\mathbf{F}\right\|_F \leq \frac{q}{\lambda_{\hat{\mathbf{D}}}} \left\|\mathbf{D}-\hat{\mathbf{D}}\right\|_F + \frac{2\sqrt{q}}{\lambda_{\hat{\mathbf{D}}}\sqrt{m}} \left\|\hat{\mathbf{D}}\right\|_F . \tag{16}$$

We can find that as the number of samples m increases, the upper bound of $\left\|\bar{\mathbf{\Delta}}_\mathbf{F}\right\|_F$ decreases, which indicates that more training samples will push the partial label matrix to be close to the ground-truth one and achieve better PLL performance. Moreover, a smaller error between $\mathbf{D}$ and $\hat{\mathbf{D}}$ implies a smaller upper bound of $\left\|\bar{\mathbf{\Delta}}_\mathbf{F}\right\|_F$, which indicates a better dissimilarity matrix can help achieve a better label matrix.

B3. Proof of Theorem 2 Inequality 2

Denote $\mathbf{\Delta_D} = \hat{\mathbf{D}} - \mathbf{D}$. According to the adversarial relationship and dissimilarity propagation of DPCLS, we assume each sample and its k neighborhoods belong to the same class, then the following inequality holds

$$\langle \mathbf{F_G}\mathbf{F_G}^\mathsf{T}, \mathbf{D}+\mathbf{\Delta_D}\rangle + \mathbf{Tr}((\mathbf{D}+\mathbf{\Delta_D})\mathbf{L}(\mathbf{D}+\mathbf{\Delta_D})^\mathsf{T}) \leq \langle \mathbf{F}\mathbf{F}^\mathsf{T}, \mathbf{D}\rangle + \mathbf{Tr}((\mathbf{D}\mathbf{L}\mathbf{D}^\mathsf{T}). \tag{17}$$

Expand Eq. (17), we have

$$\begin{aligned}\langle \mathbf{\Delta_D}^\mathsf{T}\mathbf{\Delta_D}, \mathbf{L}\rangle &\leq \langle \mathbf{F}\mathbf{F}^\mathsf{T} - \mathbf{F_G}\mathbf{F_G}^\mathsf{T}, \mathbf{D}\rangle - 2\langle \mathbf{\Delta_D}, \mathbf{D}\mathbf{L}\rangle - \langle \mathbf{F_G}\mathbf{F_G}^\mathsf{T}, \mathbf{\Delta_D}^\mathsf{T}\rangle \\ &= \langle \mathbf{F}\mathbf{F}^\mathsf{T} - \mathbf{F_G}\mathbf{F_G}^\mathsf{T}, \mathbf{D}\rangle + 2\langle \mathbf{\Delta_D}, -\mathbf{D}\mathbf{L}\rangle + \langle \mathbf{F_G}\mathbf{F_G}^\mathsf{T}, -\mathbf{\Delta_D}^\mathsf{T}\rangle \\ &\leq \left\|\mathbf{F}\mathbf{F}^\mathsf{T} - \mathbf{F_G}\mathbf{F_G}^\mathsf{T}\right\|_F \|\mathbf{D}\|_F + 2\|\mathbf{D}\|_F \|\mathbf{L}\|_F \|\mathbf{\Delta_D}\|_F + \|\mathbf{F_G}\|_F^2 \|\mathbf{\Delta_D}\| .\end{aligned} \tag{18}$$

Assume that at least one corresponding position of $\hat{\mathbf{D}}$ and $\mathbf{D}$ has different values, i.e., $\|\mathbf{\Delta_D}\|_F \geq 1$, we have

$$\langle \mathbf{\Delta_D}^\mathsf{T}\mathbf{\Delta_D}, \mathbf{L}\rangle \leq \left\|\mathbf{F}\mathbf{F}^\mathsf{T} - \mathbf{F_G}\mathbf{F_G}^\mathsf{T}\right\|_F \|\mathbf{D}\|_F \|\mathbf{\Delta_D}\|_F + 2\|\mathbf{D}\|_F \|\mathbf{L}\|_F \|\mathbf{\Delta_D}\|_F + \|\mathbf{F_G}\|_F^2 \|\mathbf{\Delta_D}\| . \tag{19}$$

Similar to **B2**, we assume the smallest eigenvalue of $\mathbf{L}$ is $\lambda_{\mathbf{L}}$ and $\lambda_{\mathbf{L}} \geq 0$, we have $\langle \mathbf{\Delta_D}^\top \mathbf{\Delta_D}, \mathbf{L} \rangle \geq \lambda_{\mathbf{L}} \|\mathbf{\Delta_D}\|_F^2$. $\|\mathbf{D}\|_F^2$ is upper bounded by m (the number of samples), i.e., $\|\mathbf{D}\|_F^2 \leq m^2$. Since $\mathbf{F_G}$ is the ground truth label matrix, we have $\|\mathbf{F_G}\|_F^2 = m$. Eq. (19) can be further relaxed as

$$\lambda_{\mathbf{L}} \|\mathbf{\Delta_D}\|_F^2 \leq m \left\|\mathbf{F}\mathbf{F}^\top - \mathbf{F_G}\mathbf{F_G}^\top\right\|_F \|\mathbf{\Delta_D}\|_F + 2m \|\mathbf{L}\|_F \|\mathbf{\Delta_D}\|_F + m \|\mathbf{\Delta_D}\|. \tag{20}$$

Let $\left\|\bar{\mathbf{\Delta}}_{\mathbf{D}}\right\|_F$ be the average distance of each corresponding position between $\hat{\mathbf{D}}$ and $\mathbf{D}$ (i.e., $\left\|\bar{\mathbf{\Delta}}_{\mathbf{D}}\right\|_F = \frac{1}{m^2}\left\|\hat{\mathbf{D}} - \mathbf{D}\right\|_F$). Dividing m^2 on both sides of Eq. (20), we finally have

$$\left\|\bar{\mathbf{\Delta}}_{\mathbf{D}}\right\|_F \leq \frac{1}{\lambda_{\mathbf{L}} m}\left\|\mathbf{F}\mathbf{F}^\top - \mathbf{F_G}\mathbf{F_G}^\top\right\|_F + \frac{2}{\lambda_{\mathbf{L}} m}\|\mathbf{L}\|_F + \frac{1}{\lambda_{\mathbf{L}} m}. \tag{21}$$

Similar to **B2**, a larger number of training samples will reduce the distance between $\mathbf{D}$ and $\hat{\mathbf{D}}$, and a smaller error between $\mathbf{F}$ and $\mathbf{F_G}$ implies a smaller upper bound of $\left\|\bar{\mathbf{\Delta}}_{\mathbf{D}}\right\|_F$, suggesting a better dissimilarity matrix.

C Details of Compared Data Sets

Table S3: Characteristics of the synthetic data sets and the real-world partial label data sets, where Avg. CLs means the average size of the candidate label set.

Type	Data set	# Examples	# Features	# Classes	Avg. CLs
Synthetic Data Set	Glass	214	9	6	-
	Steel	1941	27	7	-
	Ecoli	336	7	8	-
	Yeast	1484	8	10	-
	Optdigits	5620	64	10	-
	Usps	9298	256	10	-
	Isolet	1559	617	26	-
	Orl	400	1024	40	-
	Amazon	1500	1326	50	-
	Bookmark	2500	1413	57	-
Real-world Data Set	FG-NET	1002	262	78	7.48
	Lost	1122	108	16	2.23
	MSRCv2	1758	48	23	3.16
	BirdSong	4998	38	13	2.18
	Malagasy	5303	384	44	8.35
	Soccer Player	17472	279	171	2.09
	Yahoo! News	22991	163	219	1.91

We evaluated ten synthetic data sets and seven real-world partial label data sets from various domains, whose details are shown in Table S3. The real-world data sets are publicly available at `http://palm.seu.edu.cn/zhangml/` and `https://github.com/dhgarrette/low-resource-pos-tagging-2013`.

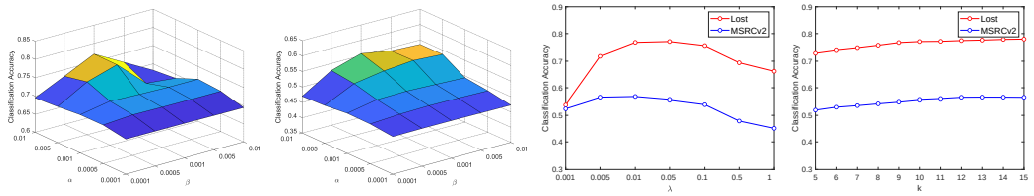
D Further Analysis

Hyper-parameters Sensitivity

Our DPCLS has four parameters, i.e., α, β, λ and k. Fig. S1 investigates their influence to DPCLS on Lost and MSRCv2. As shown in Figs. S1 (a) - (b), when α and β are too large or too small, DPCLS gives relatively poor performance. DPCLS reaches best performance when α is selected from {0.001, 0.01} and β is set to 0.001. Parameter λ controls the model complexity. We can observe from Fig. S1 (c) that the proposed model performs relatively stable when λ changes, and setting λ=0.05 is a good choice on both Lost and MSRCv2. Fig. S1 (d) indicates that the performance of DPCLS is relatively robust to different k.

Table S4: Comparison between DPCLS and DPCLS-T, where DPCLS-T indicates a two-stage model.

	FG-NET	Lost	MSRCv2	BirdSong	Malagasy	Soccer player	Yahoo! News
DPCLS	**.077±.009**	**.770±.024**	**.557±.014**	**.751±.009**	**.676±.004**	**.532±.002**	**.626±.003**
DPCLS-T	.073±.018	.712±.018•	.483±.014•	.723±.014•	.671±.005	.530±.003	.567±.003•



(a) Lost (b) MSRCv2 (c) Lost and MSRCv2 (d) Lost and MSRCv2

Figure S1: Parameter sensitivity analysis for DPCLS. (a-b) Classification accuracies of DPCLS on Lost and MSRCv2 by varying α and β; (c) Classification accuracies of DPCLS on Lost and MSRCv2 by varying λ; (d) Classification accuracies of DPCLS on Lost and MSRCv2 by varying k.

DPCLS VS. Two-stage Model

In Table S4 we compared DPCLS with a two-stage model (denoted as DPCLS-T) that performs dissimilarity matrix construction and classifier learning separately. We find that the two-stage model is significantly inferior to DPCLS, proving the advantage of end-to-end learning.

References

[1] Andreas Maurer. A vector-contraction inequality for rademacher complexities. In Ronald Ortner, Hans Ulrich Simon, and Sandra Zilles, editors, *Algorithmic Learning Theory - 27th International Conference, ALT*, volume 9925, pages 3–17, 2016.