A Related Proposition

Proposition 3 (Amoroso distribution). *The Amoroso distribution is a four parameter, continuous, univariate, unimodal probability density, with semi-infinite range [30]. And its probability density function is*

$$\mathbf{Amoroso}(X|a,\theta,\alpha,\beta) = \frac{1}{\Gamma(\alpha)}|\frac{\beta}{\theta}|(\frac{X-a}{\theta})^{\alpha\beta-1}\exp\left\{-(\frac{X-a}{\theta})^{\beta}\right\}, \tag{7}$$

for $x, a, \theta, \alpha, \beta \in \mathbb{R}, \alpha > 0$ and range $x \geq a$ if $\theta > 0$, $x \leq a$ if $\theta < 0$. The mean and variance of Amoroso distribution are

$$\mathbb{E}_{X\sim\mathbf{Amoroso}(X|a,\theta,\alpha,\beta)}X = a + \theta \cdot \frac{\Gamma(\alpha+\frac{1}{\beta})}{\Gamma(\alpha)}, \tag{8}$$

and

$$\mathbf{Var}_{X\sim\mathbf{Amoroso}(X|a,\theta,\alpha,\beta)}X = \theta^2\left[\frac{\Gamma(\alpha+\frac{2}{\beta})}{\Gamma(\alpha)} - \frac{\Gamma(\alpha+\frac{1}{\beta})^2}{\Gamma(\alpha)^2}\right]. \tag{9}$$

Proposition 4 (Half-normal distribution). *Let random variable X follow a normal distribution $N(0,\sigma^2)$, then $Y = |X|$ follows a half-normal distribution [31]. Moreover, Y also follows $\mathbf{Amoroso}(x|0,\sqrt{2}\sigma,\frac{1}{2},2)$. By Eq. (8) and Eq. (9), the mean and variance of half-normal distribution are*

$$\mathbb{E}_{X\sim N(0,\sigma^2)}|X| = \sigma\sqrt{2/\pi}, \tag{10}$$

and

$$\mathbf{Var}_{X\sim N(0,\sigma^2)}|X| = \sigma^2\left(1-\frac{2}{\pi}\right). \tag{11}$$

Proposition 5 (Scaled Chi distribution). *Let $X = (x_1, x_2, ...x_k)$ and $x_i, i = 1, ..., k$ are k independent, normally distributed random variables with mean 0 and standard deviation σ. The statistic $\ell_2(X) = \sqrt{\sum_{i=1}^k x_i^2}$ follows Scaled Chi distribution [30]. Moreover, $\ell_2(X)$ also follows $\mathbf{Amoroso}(x|0,\sqrt{2}\sigma,\frac{k}{2},2)$. By Eq. (8) and Eq. (9), the mean and variance of Scaled Chi distribution are*

$$\mathbb{E}_{X\sim N(\mathbf{0},\sigma^2\cdot\mathbf{I_k})}[\ell_2(X)]^j = 2^{j/2}\sigma^j \cdot \frac{\Gamma(\frac{k+j}{2})}{\Gamma(\frac{k}{2})}, \tag{12}$$

and

$$\mathbf{Var}_{X\sim N(\mathbf{0},\sigma^2\cdot\mathbf{I_k})}\ell_2(X) = 2\sigma^2\left[\frac{\Gamma(\frac{k}{2}+1)}{\Gamma(\frac{k}{2})} - \frac{\Gamma(\frac{k+1}{2})^2}{\Gamma(\frac{k}{2})^2}\right]. \tag{13}$$

Proposition 6 (Stirling's formula). [6] *For big enough x and $x \in \mathbb{R}^+$, we have an approximation of Gamma function:*

$$\Gamma(x+1) \approx \sqrt{2\pi x}\left(\frac{x}{e}\right)^x. \tag{14}$$

Proposition 7 (FKG inequality). *If f and g are increasing functions on $\mathbb{R}^n$ [32], we have*

$$\mathbb{E}(f)\mathbb{E}(g) \leq \mathbb{E}(fg). \tag{15}$$

Say that a function on $\mathbb{R}^n$ is increasing if it is an increasing function in each of its arguments.(i.e., for fixed values of the other arguments).

[6] `en.wikipedia.org/wiki/Stirling'sapproximation`

Proposition 8. *Let $f(X,Y)$ is a two dimensional differentiable function. According to Taylor theorem [33], we have*

$$f(X,Y) = f(\mathbb{E}(X),\mathbb{E}(Y)) + \sum_{cyc}(X-\mathbb{E}(X))\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y)) + Remainder1, \tag{16}$$

$$\begin{aligned} f(X,Y) = f(\mathbb{E}(X),\mathbb{E}(Y)) + \sum_{cyc}(X-\mathbb{E}(X))\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y)) + \\ \frac{1}{2}\sum_{cyc}(X-\mathbb{E}(X))^T\frac{\partial^2}{\partial X^2}f(\mathbb{E}(X),\mathbb{E}(Y))(X-\mathbb{E}(X)) + Remainder2 \end{aligned} \tag{17}$$

Lemma 1. *Let X and Y are random variables. Then we have such an estimation*

$$\mathbf{Var}\left(\frac{X}{Y}\right) \approx \left(\frac{\mathbb{E}(X)}{\mathbb{E}(Y)}\right)^2\left(\frac{\mathbf{Var}X}{\mathbb{E}(X)^2} + \frac{\mathbf{Var}Y}{\mathbb{E}(Y)^2} - 2\frac{\mathbf{Cov}(X,Y)}{\mathbb{E}(X)\mathbb{E}(Y)}\right). \tag{18}$$

Proof. Let $f(X,Y) = X/Y$, according to the definition of variance, we have

$$\begin{aligned} \mathbf{Var}f(X,Y) &= \mathbb{E}[f(X,Y) - \mathbb{E}(f(X,Y))]^2 \\ &\approx \mathbb{E}[f(X,Y) - \mathbb{E}\left\{f(\mathbb{E}(X),\mathbb{E}(Y)) + \sum_{cyc}(X-\mathbb{E}(X))\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y))\right\}]^2 \\ &\qquad \text{from Eq. (16)} \\ &= \mathbb{E}[f(X,Y) - f(\mathbb{E}(X),\mathbb{E}(Y)) - \sum_{cyc}\mathbb{E}(X-\mathbb{E}(X))\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y))]^2 \\ &= \mathbb{E}[f(X,Y) - f(\mathbb{E}(X),\mathbb{E}(Y))]^2 \\ &\approx \mathbb{E}[\sum_{cyc}(X-\mathbb{E}(X))\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y))]^2 \qquad \text{from Eq. (16)} \\ &= 2\mathbf{Cov}(X,Y)\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y))\frac{\partial}{\partial Y}f(\mathbb{E}(X),\mathbb{E}(Y)) + \sum_{cyc}[\frac{\partial}{\partial X}f(\mathbb{E}(X),\mathbb{E}(Y))]^2 \cdot \mathbf{Var}X \\ &= 2\mathbf{Cov}(X,Y)\cdot\frac{1}{\mathbb{E}(Y)}\cdot\left(-\frac{\mathbb{E}(X)}{(\mathbb{E}(Y))^2}\right) + \frac{1}{(\mathbb{E}(Y))^2}\cdot\mathbf{Var}X + \frac{(\mathbb{E}X)^2}{(\mathbb{E}Y)^4}\cdot\mathbf{Var}Y \\ &= \left(\frac{\mathbb{E}(X)}{\mathbb{E}(Y)}\right)^2\left(\frac{\mathbf{Var}X}{\mathbb{E}(X)^2} + \frac{\mathbf{Var}Y}{\mathbb{E}(Y)^2} - 2\frac{\mathbf{Cov}(X,Y)}{\mathbb{E}(X)\mathbb{E}(Y)}\right). \end{aligned}$$

□

From Eq.(17) and **Lemma 1**, we also can obtain an estimation of $\mathbb{E}(\mathbf{A}/\mathbf{B})$, where $\mathbf{A}$ and $\mathbf{B}$ are two random variables. *i.e.*,

$$\mathbb{E}\left(\frac{\mathbf{A}}{\mathbf{B}}\right) \approx \frac{\mathbb{E}\mathbf{A}}{\mathbb{E}\mathbf{B}} + \mathbf{Var}(\mathbf{B})\cdot\frac{\mathbb{E}\mathbf{A}}{(\mathbb{E}\mathbf{B})^3}. \tag{19}$$

Lemma 2. *For big enough x and $x \in \mathbb{R}^+$, we have*

$$\lim_{x\to+\infty}\left[\frac{\Gamma(\frac{x+1}{2})}{\Gamma(\frac{x}{2})}\right]^2\cdot\frac{1}{x} = \frac{1}{2}. \tag{20}$$

And

$$\lim_{x\to+\infty}\frac{\Gamma(\frac{x}{2}+1)}{\Gamma(\frac{x}{2})} - \left[\frac{\Gamma(\frac{x+1}{2})}{\Gamma(\frac{x}{2})}\right]^2 = \frac{1}{4}. \tag{21}$$

Proof.

$$\begin{aligned}
\lim_{x\to+\infty}\left[\frac{\Gamma(\frac{x+1}{2})}{\Gamma(\frac{x}{2})}\right]^2\cdot\frac{1}{x} &\approx \lim_{x\to+\infty}\left(\frac{\sqrt{2\pi(\frac{x-1}{2})}\cdot(\frac{x-1}{2e})^{\frac{x-1}{2}}}{\sqrt{2\pi(\frac{x-2}{2})}\cdot(\frac{x-2}{2e})^{\frac{x-2}{2}}}\right)^2\cdot\frac{1}{x} \qquad \text{from Proposition. 6}\\
&= \lim_{x\to+\infty}\left(\frac{x-1}{x-2}\right)\cdot\frac{(\frac{x-1}{2e})^{x-2}}{(\frac{x-2}{2e})^{x-2}}\cdot\left(\frac{x-1}{2e}\right)\cdot\frac{1}{x}\\
&= \lim_{x\to+\infty}\left(1+\frac{1}{x-2}\right)^{x-2}\cdot\frac{x-1}{x-2}\cdot\frac{x-1}{2e}\cdot\frac{1}{x}\\
&= \frac{1}{2}
\end{aligned}$$

on the other hand, we have

$$\begin{aligned}
\lim_{x\to+\infty}\frac{\Gamma(\frac{x}{2}+1)}{\Gamma(\frac{x}{2})}-\left[\frac{\Gamma(\frac{x+1}{2})}{\Gamma(\frac{x}{2})}\right]^2 &= \lim_{x\to+\infty}\frac{x}{2}-\left(1+\frac{1}{x-2}\right)^{x-2}\cdot\frac{x-1}{x-2}\cdot\frac{x-1}{2e}\\
&= \lim_{x\to+\infty}\frac{x}{2e}\left(e-(1+\frac{1}{x})^x\right)\\
&= \frac{1}{2}\left(-\frac{\frac{1}{e}(-e)}{2}\right)\\
&= \frac{1}{4}
\end{aligned}$$

□

Proposition 9. *KL divergence between two distributions P and Q of a continuous random variable is given by $D_{KL}(p\|q)=\int_x p(x)\log\frac{p(x)}{q(x)}$. And probabilty density function of multivariate Normal distribution is given by $p(\mathbf{x})=\frac{1}{(2\pi)^{k/2}|\Sigma|^{1/2}}\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$. Let our two Normal distributions be $\mathcal{N}\left(\boldsymbol{\mu_p},\Sigma_p\right)$ and $\mathcal{N}\left(\boldsymbol{\mu}_q,\Sigma_q\right)$, both k dimensional. we have*

$$D_{KL}(p\|q)=\frac{1}{2}\left[\log\frac{|\Sigma_q|}{|\Sigma_p|}-k+\left(\boldsymbol{\mu_p}-\boldsymbol{\mu_q}\right)^T\Sigma_q^{-1}\left(\boldsymbol{\mu_p}-\boldsymbol{\mu_q}\right)+\operatorname{tr}\left\{\Sigma_q^{-1}\Sigma_p\right\}\right]. \tag{22}$$

Proposition 10 (Jacobi's formula)**.** *If A is a differentiable map from the real numbers to $n\times n$ matrices,*

$$\frac{d}{dt}\det A(t)=\operatorname{tr}\left(\operatorname{adj}(A(t))\frac{dA(t)}{dt}\right). \tag{23}$$

Proposition 11. *For random variable X with μ and σ^2 as mean and variance, then we can use Taylor expansion to obtain:*

$$\begin{cases}\mathbb{E}(\log X)\approx\log\mu-\frac{\sigma^2}{2\mu^2}\\ \mathbf{Var}(\log X)\approx\frac{\sigma^2}{\mu^2}\end{cases}. \tag{24}$$

Proposition 12. *Given n normal distributions $N(0,\sigma_i^2), 1\le i\le n$ and $(X_{i1},X_{i2},...,X_{im})$ are sample from $N(0,\sigma_i^2), 1\le j\le m$. then*

$$\mathbf{Var}_{1\le i\le n,1\le j\le m}(X_{ij})=\frac{1}{n}\sum_{i=1}^{n}\sigma_i^2. \tag{25}$$

Proof.

$$\mathbf{Var}_{1\le i\le n,1\le j\le m}(X_{ij}) = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}[X_{ij}-\mathbb{E}(X_{ij})]^2 \tag{26}$$

$$= \frac{1}{n}\{\frac{1}{m}\sum_{j=1}^{m}[X_{ij}-\mathbb{E}(X_{1j})]^2 + ... + \frac{1}{m}\sum_{j=1}^{m}[X_{nj}-\mathbb{E}(X_{nj})]^2\}$$

Since $\mathbb{E}(X_{ij}) = 0, 1 \le i \le n, 1 \le j \le m$

$$= \frac{1}{n}\{\sigma_1^2 + ... + \sigma_n^2\} \tag{27}$$

□

Lemma 3. *For a matrix $\mathbf{B} \in R^{n\times n}$ and a small constant ϵ, we have:*

$$det(\mathbf{I}_n + \epsilon\mathbf{B}) = 1 + \epsilon\,\mathrm{tr}(\mathbf{B}) + O(\epsilon^2). \tag{28}$$

Proof. First, we regard $det(\mathbf{I}_n + \epsilon\mathbf{B})$ as a function w.r.t ϵ. Since Proposition 10, we have:

$$\frac{d}{d\epsilon}det(\mathbf{I}_n + \epsilon\mathbf{B})|_{\epsilon=0} = \mathrm{tr}\{\mathrm{adj}(\mathbf{I}_n + \epsilon\mathbf{B})\mathbf{B}\}|_{\epsilon=0} \tag{29}$$

$$= \mathrm{tr}\{det(\mathbf{I}_n + \epsilon\mathbf{B}) \cdot (\mathbf{I}_n + \epsilon\mathbf{B})^{-1}\mathbf{B}\}|_{\epsilon=0} \tag{30}$$

$$= det(\mathbf{I}_n + \epsilon\mathbf{B}) \cdot \mathrm{tr}\{(\mathbf{I}_n + \epsilon\mathbf{B})^{-1}\mathbf{B}\}|_{\epsilon=0} \tag{31}$$

$$= \mathrm{tr}(\mathbf{B}) \tag{32}$$

Using Taylor expansion for $det(\mathbf{I}_n+\epsilon\mathbf{B})$, we have $\frac{d}{d\epsilon}det(\mathbf{I}_n+\epsilon\mathbf{B}) = det(\mathbf{I}_n) + \frac{d}{d\epsilon}det(\mathbf{I}_n+\epsilon\mathbf{B})|_{\epsilon=0}\cdot \epsilon + O(\epsilon^2)$. In other words, $det(\mathbf{I}_n + \epsilon\mathbf{B}) = 1 + \epsilon\,\mathrm{tr}(\mathbf{B}) + O(\epsilon^2)$.

□

A.1 The proof of Proposition 1

(**Proposition** 1) If the convolutional filters F_A in layer A meet CWDA, then we have following estimations:

Criterion	Mean	Variance
$\ell_1(F_A)$	$\sqrt{2/\pi}\sigma_A d_A$	$(1-\frac{2}{\pi})\sigma_A^2 d_A$
$\ell_2(F_A)$	$\sqrt{2}\sigma_A\Gamma(\frac{d_A+1}{2})/\Gamma(\frac{d_A}{2})$	$\sigma_A^2/2$
$\mathbf{Fermat}(F_A)$	$\sqrt{2}\sigma_A\Gamma(\frac{d_A+1}{2})/\Gamma(\frac{d_A}{2})$	$\sigma_A^2/2$

where d_A and σ_A^2 denote the dimension of F_A and the variance of the weights in layer A, respectively.

Proof. According to Appendix B, Eq. (21), Proposition 4 and Proposition 5, we can obtain the mean and variance of $\ell_1(F_A)$ and $\ell_2(F_A)$. Moreover, From the Theorem 3, we know that the Fermat point $\mathbf{F}$ of F_A and the origin $\mathbf{0}$ approximately coincide. According to Table 1, $||\mathbf{F} - F_A||_2 \approx ||\mathbf{0} - F_A||_2 = ||F_A||_2$. Therefore, the mean and variance of $\mathbf{Fermat}(F_A)$ are the same as $\ell_2(F_A)$'s in Proposition 1.

□

A.2 The proof of Proposition 2

(**Proposition** 2) If the convolutional filters F_A in layer A meet CWDA, then $\mathbb{E}[\ell_1(F_A)/\ell_2(F_A)]$ and $\mathbb{E}[\ell_2(F_A)/\ell_1(F_A)]$ only depend on their dimension d_A.

Proof. From Eq. (19), we have:

$$\begin{aligned}
\mathbb{E}[\frac{\ell_1(F_A)}{\ell_2(F_A)}] &\approx \frac{\mathbb{E}[\ell_1(F_A)]}{\mathbb{E}[\ell_2(F_A)]} + \mathbf{Var}[\ell_2(F_A)] \cdot \frac{\mathbb{E}[\ell_1(F_A)]}{\mathbb{E}[\ell_2(F_A)]^3} \\
&= \frac{\sqrt{2/\pi}\sigma_A d_A}{\sqrt{2}\sigma_A \Gamma(\frac{d_A+1}{2})/\Gamma(\frac{d_A}{2})} + \sigma_A^2/2 \cdot \frac{\sqrt{2/\pi}\sigma_A d_A}{[\sqrt{2}\sigma_A \Gamma(\frac{d_A+1}{2})/\Gamma(\frac{d_A}{2})]^3} && \text{from Proposition. 1} \\
&\approx O(\sqrt{d_A}) + O(\frac{1}{\sqrt{d_A}}) && \text{from Eq. (20)}
\end{aligned}$$

Similarly, we can prove that $\mathbb{E}[\ell_2(F_A)/\ell_1(F_A)]$ also only depend on their dimension d_A.

$$\begin{aligned}
\mathbb{E}[\frac{\ell_2(F_A)}{\ell_1(F_A)}] &\approx \frac{\mathbb{E}[\ell_2(F_A)]}{\mathbb{E}[\ell_1(F_A)]} + \mathbf{Var}[\ell_1(F_A)] \cdot \frac{\mathbb{E}[\ell_2(F_A)]}{\mathbb{E}[\ell_1(F_A)]^3} \\
&= \frac{\sqrt{2}\sigma_A \Gamma(\frac{d_A+1}{2})/\Gamma(\frac{d_A}{2})}{\sqrt{2/\pi}\sigma_A d_A} + (1 - \frac{2}{\pi})\sigma_A^2 d_A \cdot \frac{\sqrt{2}\sigma_A \Gamma(\frac{d_A+1}{2})/\Gamma(\frac{d_A}{2})}{[\sqrt{2/\pi}\sigma_A d_A]^3} \\
& && \text{from Proposition. 1} \\
&\approx O(\frac{1}{\sqrt{d_A}}) + O(\frac{1}{d_A^{1.5}}) && \text{from Eq. (20)}
\end{aligned}$$

□

B The relaxation for CWDA

(Convolution Weight Distribution Assumption) Let $F_{ij} \in \mathbb{R}^{N_i \times k \times k}$ be the j^{th} well-trained filter of the i^{th} convolutional layer. In general[7], in i^{th} layer, F_{ij} $(j = 1, 2, ..., N_{i+1})$ are i.i.d and follow such a distribution:

$$F_{ij} \sim \mathbf{N}(\mathbf{0}, \mathbf{\Sigma}^i_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}^i_{\text{block}}), \tag{33}$$

where $\mathbf{\Sigma}^i_{\text{block}} = \text{diag}(K_1, K_2, ..., K_{N_i})$ is a block diagonal matrix and the diagonal elements of $\mathbf{\Sigma}^i_{\text{block}}$ are 0. ϵ is a small constant. The values of the off-block-diagonal elements are 0 and $K_l \in R^{k^2 \times k^2}, l = 1, 2, ..., N_i$. $\mathbf{\Sigma}^i_{\text{diag}} = \text{diag}(a_1, a_2, ..., a_{N_i \times k \times k})$ is a diagonal matrix and the elements of $\mathbf{\Sigma}^i_{\text{diag}}$ are close enough.

In Section 2, we propose CWDA. In order to use this assumption conveniently, we give the following relaxation of CWDA:

(**Convolution Weight Distribution Assumption-Relaxation**) Let $F_{ij} \in \mathbb{R}^{N_i \times k \times k}$ be the j^{th} well-trained filter of the i^{th} convolutional layer. In general, in i^{th} layer, F_{ij} $(j = 1, 2, ..., N_{i+1})$ are i.i.d and follow such a distribution:

$$F_{ij} \sim \mathbf{N}(\mathbf{0}, \sigma^2_{\text{layer}} \cdot \mathbf{I}_{N_i \times k \times k}), \tag{34}$$

where σ^2_{layer} is the variance of the weights in i^{th} convolutional layer.

Next, we analyze the gap between CWDA and CWDA-Relaxation, *i.e.,* the difference between $\mathbf{N}(\mathbf{0}, \mathbf{\Sigma}^i_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}^i_{\text{block}})$ and $\mathbf{N}(\mathbf{0}, \sigma^2_{\text{layer}} \cdot \mathbf{I}_{N_i \times k \times k})$.

Lemma 4. *Given two n-dimension Gaussian distributions $\mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{diag} + \epsilon \cdot \mathbf{\Sigma}_{block})$ and $\mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{diag})$, we can estimate the KL divergence of them:*

$$\text{KL}[\mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{diag} + \epsilon \cdot \mathbf{\Sigma}_{block}) || \mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{diag})] \approx \frac{1}{2} \log[\frac{1}{1 + O(\epsilon^2)}] \tag{35}$$

[7]In Section 6, we make further discussion and analysis on the conditions for CWDA to be satisfied.

where $\mathbf{\Sigma}_{block} = \operatorname{diag}(K_1, K_2, ..., K_{N_i})$ *is a block diagonal matrix and the diagonal elements of* $\mathbf{\Sigma}_{block}$ *are 0.* ϵ *is a small constant. The values of the off-block-diagonal elements are 0 and* $K_l \in R^{k^2 \times k^2}, l = 1, 2, ..., N_i$. $\mathbf{\Sigma}_{diag} = \operatorname{diag}(a_1, a_2, ..., a_{N_i \times k \times k})$ *is a diagonal matrix and the elements of* $\mathbf{\Sigma}_{diag}$ *are close enough.* $n = N_i \times k \times k$.

Proof. Since Proposition 9, we have:

$$2\,\mathrm{KL} = \log \frac{det[\mathbf{\Sigma}_{\text{diag}}]}{det[\mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}}]} - n + 0 + \operatorname{tr}\{\mathbf{\Sigma}_{\text{diag}}^{-1}(\mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}})\} \tag{36}$$

$$= \log \frac{det[\mathbf{\Sigma}_{\text{diag}}]}{det[\mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}}]} - n + \operatorname{tr}\{\mathbf{I}_k + \epsilon \mathbf{\Sigma}_{\text{diag}}^{-1} \mathbf{\Sigma}_{\text{block}}\} \tag{37}$$

$$= \log \frac{det[\mathbf{\Sigma}_{\text{diag}}]}{det[\mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}}]} \qquad \text{Since the diagonal elements of } \mathbf{\Sigma}_{\text{block}} \text{ are 0} \tag{38}$$

Let $\mathbf{\Sigma}_{\text{diag}} = \operatorname{diag}(S_1, S_2, ..., S_{N_i})$, where $S_j = \operatorname{diag}(a_{(j-1)k^2+1}, a_{(j-1)k^2+2}, ..., a_{(j-1)k^2+k^2}), j = 1, 2, ..., N_i$.

$$2\,\mathrm{KL} = \log \frac{det[\mathbf{\Sigma}_{\text{diag}}]}{det[\mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}}]} \tag{39}$$

$$= \log \prod_{j=1}^{n} a_k - \log\{\prod_{h=1}^{N_i} det[S_h + \epsilon K_h]\} \tag{40}$$

$$= \log \prod_{j=1}^{n} a_k - \log\{\prod_{h=1}^{N_i} det[S_h] det[\mathbf{I}_{k^2} + \epsilon S_h^{-1} K_h]\} \qquad \text{Since } S_h \succeq 0 \tag{41}$$

Note that S_h is a diagonal matrix and the diagonal elements of K_h are all zero. Therefore

$$\operatorname{tr}(S_h^{-1} K_h) = 0. \tag{42}$$

Next,

$$2\,\mathrm{KL} = \log \prod_{j=1}^{n} a_k - \log\{\prod_{h=1}^{N_i} det[S_h] det[\mathbf{I}_{k^2} + \epsilon S_h^{-1} K_h]\} \tag{43}$$

$$= \log \prod_{j=1}^{n} a_k - \log\{\prod_{h=1}^{N_i} det[S_h] \cdot (1 + \epsilon \operatorname{tr}(S_h^{-1} K_h) + O(\epsilon^2))\} \qquad \text{Since Lemma 3}$$

$$= \log \prod_{j=1}^{n} a_k - \log\{\prod_{h=1}^{N_i} det[S_h] \cdot (1 + O(\epsilon^2))\} \qquad \text{Since Eq. (42)}$$

$$= \log \prod_{j=1}^{n} a_k - \log \prod_{j=1}^{n} a_k (1 + O(\epsilon^2)) \tag{44}$$

$$= \log[\frac{1}{1 + O(\epsilon^2)}] \tag{45}$$

□

According to Statistical test (2) in Section 2.1, $\mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{\text{diag}})$ can be approximate to $\mathbf{N}(\mathbf{0}, \frac{1}{n} \operatorname{tr}(\mathbf{\Sigma}_{\text{diag}}) \mathbf{I}_n)$. In addition, from Propsition 12 and Lemma 4, while ϵ is small enough, the distribution $\mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}})$ can be approximate to $\mathbf{N}(\mathbf{0}, \sigma_{\text{layer}}^2 \cdot \mathbf{I}_{N_i \times k \times k})$. The analysis in this paper are based on *Convolution Weight Distribution Assumption-Relaxation* and we use it to explain successfully many phenomena in the Similarity and Applicability problem of pruning criteria.

C Proof of Theorem 1

Theorem 1. Let $n-$dimension random variable X meet CWDA, and the pair of criteria (C_1, C_2) is one of (ℓ_1, ℓ_2), $(\ell_2, \mathbf{Fermat})$ or $(\mathbf{Fermat}, \mathbf{GM})$, we have

$$\mathbf{max}\left\{\mathbf{Var}_X\left(\frac{\widehat{C}_2(X)}{\widehat{C}_1(X)}\right), \mathbf{Var}_X\left(\frac{\widehat{C}_1(X)}{\widehat{C}_2(X)}\right)\right\} \lesssim B(n). \tag{46}$$

where $\widehat{C}_1(X)$ denotes $C_1(X)/\mathbb{E}(C_1(X))$ and $\widehat{C}_2(X)$ denotes $C_2(X)/\mathbb{E}(C_2(X))$. $B(n)$ denotes the upper bound of left-hand side and when n is large enough, $B(n) \to 0$.

For i^{th} layer, we use v_j to represent F_{ij}, $j = 1, 2, ...N$. And v_j meets CWDA. Since Appendix B, we use the following three points to prove Theorem 1.

(1) For (ℓ_2, ℓ_1). In fact, $\ell_2 \cong \ell_1$ (their importance rankings are similar) is not trivial. Generally speaking, for convolutional filters, $\mathbf{dim}(v_j)$ is large enough. Since v_i satisfies CWDA, from Theorem 2, we know that the variance of ratio between $\widehat{\ell_1}$ and $\widehat{\ell_2}$ have a bound $O(\mathbf{dim}(v_j)^{-1})$, which means ℓ_2 and ℓ_1 are *appropriate monotonic*. Specific numerical validation is shown in Fig. 9 of Appendix D).

Theorem 2. *Let $X \sim N(\mathbf{0}, c^2 \cdot \mathbf{I}_n)$, we have*

$$\mathbf{max}\left\{\mathbf{Var}_X\left(\frac{\widehat{\ell}_2(X)}{\widehat{\ell}_1(X)}\right), \mathbf{Var}_X\left(\frac{\widehat{\ell}_1(X)}{\widehat{\ell}_2(X)}\right)\right\} \lesssim \frac{1}{n}. \tag{47}$$

where $\widehat{\ell}_1(X)$ denotes $\ell_1(X)/\mathbb{E}(\ell_1(X))$ and $\widehat{\ell}_2(X)$ denotes $\ell_2(X)/\mathbb{E}(\ell_2(X))$. c is a constant.

Proof. (See Appendix D). □

(2) For $(\ell_2, \mathbf{Fermat})$. Since v_i satisfies CWDA, from Theorem 3, we know that the Fermat point of v_i and the origin $\mathbf{0}$ approximately coincide. According to Table 2, $||\mathbf{Fermat} - v_i||_2 \approx ||\mathbf{0} - v_i||_2 = ||v_i||_2$. Therefore, from Theorem 2, the bound $B(n)$ for the $(\ell_1, \mathbf{Fermat})$ and $(\ell_2, \mathbf{Fermat})$ are $\frac{1}{n}$ and 0, respectively. Moreover, since CWDA, the centroid of v_i is $\mathbf{G} = \frac{1}{n}\sum_{i=1}^{N} v_i = \mathbf{0}$. Hence,

$$\mathbf{G} = \mathbf{0} \approx \mathbf{Fermat}. \tag{48}$$

Theorem 3. *Let random variable $v_i \in \mathbb{R}^k$ and they are i.i.d and follow normal distribution $N(\mathbf{0}, \sigma^2\mathbf{I}_k)$. For $F \in \mathbb{R}^k$, we have $\mathbf{argmin}_F \left\{\mathbb{E}_{v_i \sim N(\mathbf{0}, \sigma^2\mathbf{I}_k)} \sum_{i=1}^{n} ||F - v_i||_2\right\} = \mathbf{0}$.*

Proof. (See Appendix E). □

(3) For $(\mathbf{GM}, \mathbf{Fermat})$. First, we show the following two theorems:

Theorem 4. *For n random variables $a_i \in \mathbb{R}^k$ follow $N(\mathbf{0}, c^2 \cdot \mathbf{I}_k)$.When k is large enough, we have such an estimation:*

$$\mathbf{Var}_{a_i} \frac{F_1(a_i)}{F_2(a_i)} \approx \frac{1}{2nk}, \quad \mathbf{Var}_{a_i} \frac{F_2(a_i)}{F_1(a_i)} \approx \frac{1}{2nk}, \tag{49}$$

where $F_1(a_i) = \sum_{i=1}^{n} ||a_i||_2/\mathbb{E}(\sum_{i=1}^{n} ||a_i||_2)$ and $F_2(a_i) = \sum_{i=1}^{n} ||a_i||_2^2/\mathbb{E}(\sum_{i=1}^{n} ||a_i||_2^2)$.

Proof. (See Appendix F). □

Theorem 5. *Let $v_0, v_1, ..., v_k$ be the $k+1$ vectors in n dimensional Euclidean space $\mathbb{E}^n$. For all P in $\mathbb{E}^n$,*

$$\sum_{i=0}^{k} ||P - v_i||_2^2 = \sum_{i=0}^{k} ||G - v_i||_2^2 + (k+1)||P - G||_2^2, \tag{50}$$

where G is the centroid of v_i, will hold if it satisfies one of the following conditions:

(1)if $k \geq n$ and $\mathbf{rank}(v_1 - v_0, v_2 - v_0, ..., v_k - v_0) = n$.

(2)if $k < n$ and $(v_1 - v_0, v_2 - v_0, ..., v_k - v_0)$ are linearly independent.

(3)if $v_i \sim N(\mathbf{0}, c^2 \cdot \mathbf{I}_n)$, Eq.(50) holds with probability 1.

Proof. (See Appendix G). □

Let $P \in \{v_1, v_2, ..., v_N\}$. Since $v_i \sim N(\mathbf{0}, c^2 \cdot \mathbf{I})$, we can obtain that $a_i = P - v_i \sim N(\mathbf{0}, 2c^2 \cdot \mathbf{I})$ if $P \neq v_i$. According to the analysis in Section 3.1 and Theorem 4, we have

$$\sum_{i=1}^{n} ||a_i||_2 \cong \sum_{i=1}^{n} ||a_i||_2^2, \tag{51}$$

Next, we can prove $(k+1)||P-F||_2^2$ (**Fermat**) and $\sum_{i=1}^{N} ||P - v_i||_2$ (**GM**) are *approximately monotonic*, where $P \in \{v_1, v_2, ..., v_N\}$.

$$\begin{aligned}
(k+1)||P-F||_2^2 &\cong (k+1)||P-G||_2^2 && \text{Since Eq. (48)} \\
&= \sum_{i=1}^{N} ||P-v_i||_2^2 - \sum_{i=1}^{N} ||G-v_i||_2^2 && \text{Since Theorem 5} \\
&\cong \sum_{i=1}^{N} ||P-v_i||_2 - \sum_{i=1}^{N} ||G-v_i||_2^2 && \text{Since Eq. (51)} \\
&\cong \sum_{i=1}^{N} ||P-v_i||_2 && (52)
\end{aligned}$$

The reason for the last equation is that $\sum_{i=1}^{N} ||G - v_i||_2^2$ is a constant for given v_i.

D Proof of Theorem 2

Theorem 2 Let $X \sim N(\mathbf{0}, c^2 \cdot \mathbf{I}_n)$, we have

$$\mathbf{max}\left\{\mathbf{Var}_X\left(\frac{\widehat{\ell_2}(X)}{\widehat{\ell_1}(X)}\right), \mathbf{Var}_X\left(\frac{\widehat{\ell_1}(X)}{\widehat{\ell_2}(X)}\right)\right\} \lesssim \frac{1}{n}.$$

where $\widehat{\ell_1}(X)$ denotes $\ell_1(X)/\mathbb{E}(\ell_1(X))$ and $\widehat{\ell_2}(X)$ denotes $\ell_2(X)/\mathbb{E}(\ell_2(X))$.

Proof. For the ratio $\widehat{\ell_2}(X)/\widehat{\ell_1}(X)$, we have

$$\begin{aligned}
\mathbf{Var}\left(\frac{\widehat{\ell_2}(X)}{\widehat{\ell_1}(X)}\right) &= \left(\frac{\mathbb{E}(\ell_1(X))}{\mathbb{E}(\ell_2(X))}\right)^2 \mathbf{Var}\left(\frac{\ell_2(X)}{\ell_1(X)}\right) \\
&\approx \left(\frac{\mathbb{E}(\ell_1(X))}{\mathbb{E}(\ell_2(X))}\right)^2 \left(\frac{\mathbb{E}(\ell_2(X))}{\mathbb{E}(\ell_1(X))}\right)^2 \left(\frac{\mathbf{Var}\ell_2(X)}{\mathbb{E}(\ell_2(X))^2} + \frac{\mathbf{Var}\ell_1(X)}{\mathbb{E}(\ell_1(X))^2} - 2\frac{\mathbf{Cov}(\ell_2(X), \ell_1(X))}{\mathbb{E}(\ell_2(X))\mathbb{E}(\ell_1(X))}\right) && \text{from Lemma. 1} \\
&\leq \left(\frac{\mathbf{Var}\ell_2(X)}{\mathbb{E}(\ell_2(X))^2} + \frac{\mathbf{Var}\ell_1(X)}{\mathbb{E}(\ell_1(X))^2}\right). && \text{from Proposition. 7}
\end{aligned}$$

similarly, we also have

$$\mathbf{Var}\left(\frac{\widehat{\ell_1}(X)}{\widehat{\ell_2}(X)}\right) \leq \left(\frac{\mathbf{Var}\ell_2(X)}{\mathbb{E}(\ell_2(X))^2} + \frac{\mathbf{Var}\ell_1(X)}{\mathbb{E}(\ell_1(X))^2}\right). \tag{53}$$

Therefore,

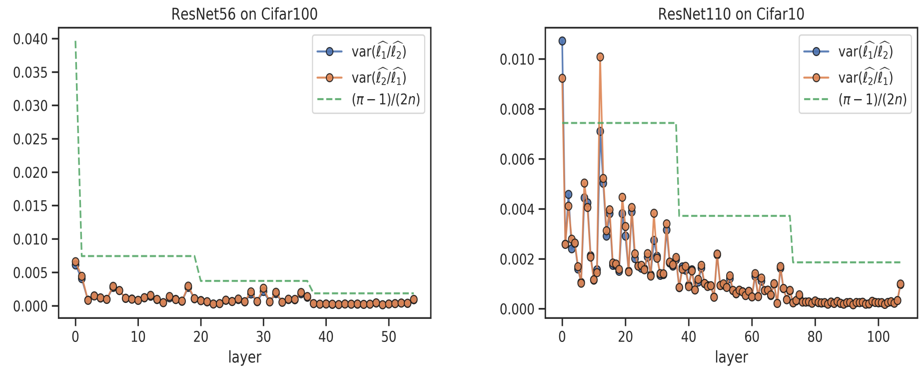


Figure 9: The approximation of **Theorem 2**: (Left) the example about ResNet56; (Right) the example about ResNet110.

$$\begin{aligned}
\mathbf{max}\left\{\mathbf{Var}_X\left(\frac{\widehat{\ell}_2(X)}{\widehat{\ell}_1(X)}\right), \mathbf{Var}_X\left(\frac{\widehat{\ell}_1(X)}{\widehat{\ell}_2(X)}\right)\right\} &\leq \left(\frac{\mathbf{Var}\ell_2(X)}{\mathbb{E}(\ell_2(X))^2} + \frac{\mathbf{Var}\ell_1(X)}{\mathbb{E}(\ell_1(X))^2}\right) \\
&= \frac{2\sigma^2\left[\frac{\Gamma(\frac{n}{2}+1)}{\Gamma(\frac{n}{2})} - \frac{\Gamma(\frac{n+1}{2})^2}{\Gamma(\frac{n}{2})^2}\right]}{(\sqrt{2}\sigma\cdot\frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})})^2} + \frac{\sigma^2\left(1-\frac{2}{\pi}\right)n}{(n\cdot\sigma\sqrt{2/\pi})^2} \\
&\qquad \text{from Proposition. 5 and 4} \\
&\approx \left(\frac{1}{2n} + (\frac{\pi}{2}-1)\frac{1}{n}\right) \qquad \text{from Lemma 2} \\
&= \frac{\pi-1}{2n}
\end{aligned}$$

□

Because the approximation is widely used in the proof of Theorem 1, it is necessary to verify it numerically. As shown in Fig. 9, we use ResNet56 on Cifar100 and ResNet110 on Cifar10 respectively to verify Theorem 1. From Fig. 9, we find that the estimationn of Theorem 1 is reliable, *i.e.*, the estimation $O(\frac{1}{n})$ for $\mathbf{max}\left\{\mathbf{Var}_X\left(\frac{\widehat{\ell}_2(X)}{\widehat{\ell}_1(X)}\right), \mathbf{Var}_X\left(\frac{\widehat{\ell}_1(X)}{\widehat{\ell}_2(X)}\right)\right\}$ is appropriate.

E Proof of Theorem 3

Proposition 13. *Let $L_p^{(\alpha)}(x)$ denotes generalized Laguerre function, and it have following properties:*

$$\frac{\partial^n}{\partial x^n}L_p^{(\alpha)} = (-1)^n L_{p-n}^{(\alpha+n)}(x), \tag{54}$$

and for $\alpha > 0$,

$$L_{-\frac{1}{2}}^{(\alpha)}(x) > 0. \tag{55}$$

Theorem 3. Let random variable $v_i \in \mathbb{R}^k$. They are i.i.d and follow normal distribution $N(\mathbf{0}, \sigma^2\mathbf{I}_k)$. For F in $\mathbb{R}^k$, we have

$$\mathbf{argmin}_F\left\{\mathbb{E}_{v_i\sim N(\mathbf{0},\sigma^2\mathbf{I}_k)}\sum_{i=1}^{n}||F-v_i||_2\right\} = \mathbf{0}.$$

Proof. Let $w_i = F - v_i$ and we have $w_i \sim N(F, \sigma^2 \mathbf{I}_k)$, then

$$\begin{aligned}\mathbb{E}_{v_i \sim N(\mathbf{0},\sigma^2\mathbf{I}_k)} \sum_{i=1}^{n} ||F - v_i||_2 &= \sum_{i=1}^{n} \mathbb{E}_{v_i \sim N(\mathbf{0},\sigma^2\mathbf{I}_k)} ||F - v_i||_2 \\ &= \sum_{i=1}^{n} \mathbb{E}_{w_i \sim N(F,\sigma^2\mathbf{I}_k)} ||w_i||_2 \\ &= n \cdot \sigma^2 \sqrt{\frac{\pi}{2}} \cdot L_{\frac{1}{2}}^{(\frac{k}{2}-1)} \left(-\frac{||F||_2^2}{2\sigma^2}\right)\end{aligned}$$

The reason for the last equation is that $||w_i||_2$ follows scaled noncentral chi distribution[8] when $w_i \sim N(F, \sigma^2 \mathbf{I}_k)$. Let $T(x) = L_{\frac{1}{2}}^{(\frac{k}{2}-1)} \left(-\frac{x^2}{2\sigma^2}\right)$, we calculate the minimum of $T(x)$. From Eq. (54),

$$\frac{d}{dx} T(x) = \frac{x}{\sigma^2} \cdot L_{-\frac{1}{2}}^{(\frac{k}{2})} \left(-\frac{x^2}{2\sigma^2}\right). \tag{56}$$

Since Eq. (55), we find that $\frac{d}{dx}T(x) > 0$ when $x > 0$ and if $x \leq 0$, then $\frac{d}{dx}T(x) \leq 0$. It means that $T(x)$ gets the minimizer at $||F||_2 = 0$, *i.e.*, $F = \mathbf{0}$.

□

F Proof of Theorem 4

Lemma 5. *For two random variables $X, Y \in \mathbb{R}^k$ follow $N(\mathbf{0}, c^2 \cdot \mathbf{I}_k)$ and they are i.i.d. When k is large enough, we have:*

$$\mathbb{E}\left(\frac{(||X||_2^2 - ||Y||_2^2)^2}{2||X||_2 \cdot ||Y||_2}\right) \approx 2c^2 + \frac{4c^2k + 1}{2k^2}, \tag{57}$$

and

$$\mathbf{Var}\left(\frac{(||X||_2^2 - ||Y||_2^2)^2}{2||X||_2 \cdot ||Y||_2}\right) \lesssim 8c^4 + \frac{16c^4k + c^2}{k^2}, \tag{58}$$

Proof. According to **Proposition 3** and **Lemma 2**, it is easy to know, when k is large enough, that

$$\mathbb{E}\left(2||X||_2 \cdot ||Y||_2\right) = 2c^2k, \quad \mathbf{Var}\left(2||X||_2 \cdot ||Y||_2\right) = c^2 + 4c^4k, \tag{59}$$

and

$$\mathbb{E}\left((||X||_2^2 - ||Y||_2^2)^2\right) = 4c^4k, \quad \mathbf{Var}\left((||X||_2^2 - ||Y||_2^2)^2\right) = 16c^8(2k^2 + 3k). \tag{60}$$

Since Lemma 1, we have an estimation

$$\begin{aligned}\mathbf{Var}\left(\frac{(||X||_2^2 - ||Y||_2^2)^2}{2||X||_2 \cdot ||Y||_2}\right) &\leq \left(\frac{\mathbb{E}(||X||_2^2 - ||Y||_2^2)^2}{\mathbb{E}2||X||_2 \cdot ||Y||_2}\right)^2 \left(\frac{\mathbf{Var}(||X||_2^2 - ||Y||_2^2)^2}{\mathbb{E}(||X||_2^2 - ||Y||_2^2)^2} + \frac{\mathbf{Var}(2||X||_2 \cdot ||Y||_2)^2)}{\mathbb{E}(2||X||_2 \cdot ||Y||_2)^2}\right) \\ &\approx \left(\frac{4c^4k}{2c^2k}\right)^2 \cdot \left(\frac{c^2 + 4c^4k}{4c^4k} + \frac{16c^8(2k^2 + 3k)}{16c^8k^2}\right) \\ &\qquad \text{Since Eq.(59) and Eq.(60)} \\ &= 8c^4 + \frac{16c^4k + c^2}{k^2}.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathbb{E}\left(\frac{(||X||_2^2 - ||Y||_2^2)^2}{2||X||_2 \cdot ||Y||_2}\right) &\approx \frac{\mathbb{E}(||X||_2^2 - ||Y||_2^2)^2}{\mathbb{E}2||X||_2 \cdot ||Y||_2} + \mathbf{Var}(2||X||_2 \cdot ||Y||_2) \cdot \frac{\mathbb{E}(||X||_2^2 - ||Y||_2^2)^2}{(\mathbb{E}2||X||_2 \cdot ||Y||_2)^3} \\ &\qquad \text{Since Eq.(19)} \\ &\approx \frac{4c^4k}{2c^2k} + \frac{4c^4k}{8c^6k^3} \cdot (c^2 + 4c^4k) \qquad \text{Since Eq.(59) and Eq.(60)} \\ &= 2c^2 + \frac{4c^2k + 1}{2k^2}.\end{aligned}$$

□

[8] Survey of simple,continuous,uniariate probability distribution and Wikipredia.

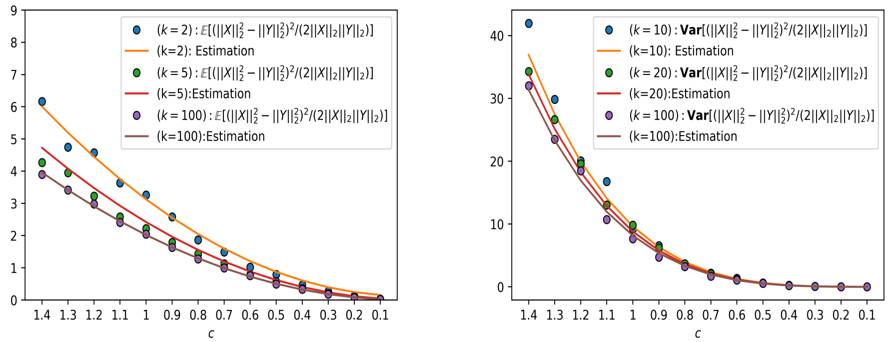


Figure 10: (Left) The numerical verification of Eq.(57) and (Right) The numerical verification of Eq.(58). X and Y follow $N(\mathbf{0}, c^2 \cdot I_k)$.

Note that, the approximation is widely used in the proof of Eq.(57) and Eq.(58). Hence, it is also necessary to verify it numerically. As shown in Fig. 10, the estimation is appropriate. According to **Lemma** 5, the mathematical expectation and variance of the ratio of $(||X||_2^2 - ||Y||_2^2)^2$ and $2||X||_2 \cdot ||Y||_2$ are both close to 0 when k is large enough and c is small enough. that is,

$$2(||X||_2 \cdot ||Y||_2) \gg (||X||_2^2 - ||Y||_2^2)^2. \tag{61}$$

By the way, the convolutional filters easily meet the condition that k is large enough.

Theorem 4. For n random variables $a_i \in \mathbb{R}^k$ follow $N(\mathbf{0}, c^2 \cdot \mathbf{I}_k)$.When k is large enough, we have such an estimation:

$$\mathbf{Var}_{a_i} \frac{F_1(a_i)}{F_2(a_i)} \approx \frac{1}{2nk}, \quad \mathbf{Var}_{a_i} \frac{F_2(a_i)}{F_1(a_i)} \approx \frac{1}{2nk}.$$

where $F_1(a_i) = \sum_{i=1}^n ||a_i||_2 / \mathbb{E}(\sum_{i=1}^n ||a_i||_2)$ and $F_2(a_i) = \sum_{i=1}^n ||a_i||_2^2 / \mathbb{E}(\sum_{i=1}^n ||a_i||_2^2)$.

Proof. Since Eq. (12) and Eq. (13), we have

$$\mathbf{Var}_{a_i} \frac{F_1(a_i)}{F_2(a_i)} = \left(\frac{nc^2k}{nc\sqrt{k}}\right)^2 \cdot \mathbf{Var}_{a_i} \left(\frac{\sum_{i=1}^n ||a_i||_2}{\sum_{i=1}^n ||a_i||_2^2}\right). \tag{62}$$

and

$$\mathbf{Var}_{a_i} \frac{F_2(a_i)}{F_1(a_i)} = \left(\frac{nc\sqrt{k}}{nc^2k}\right)^2 \cdot \mathbf{Var}_{a_i} \left(\frac{\sum_{i=1}^n ||a_i||_2^2}{\sum_{i=1}^n ||a_i||_2}\right). \tag{63}$$

According to Lagrange's identity, we have

$$\begin{aligned}
\left(\sum_{i=1}^n ||a_i||_2^2\right)\left(\sum_{i=1}^n 1\right) &= \left(\sum_{i=1}^n ||a_i||_2\right)^2 + \sum_{1\le i<j\le n} (||a_i||_2^2 - ||a_j||_2^2)^2 \\
&= \sum_{i=1}^n ||a_i||_2^2 + \sum_{1\le i<j\le n} (||a_i||_2 \cdot ||a_j||_2) + 2 \sum_{1\le i<j\le n} (||a_i||_2^2 - ||a_j||_2^2)^2 \\
&\approx \sum_{i=1}^n ||a_i||_2^2 + 2 \sum_{1\le i<j\le n} (||a_i||_2 \cdot ||a_j||_2) && \text{Since Eq. (61)} \\
&= \left(\sum_{i=1}^n ||a_i||_2\right)^2
\end{aligned}$$

so we have

$$\mathbf{Var}_{a_i \sim N(\mathbf{0}, c^2 \cdot \mathbf{I}_k)} \frac{\sum_{i=1}^n ||a_i||_2}{\sum_{i=1}^n ||a_i||_2^2} \approx \mathbf{Var}_{a_i \sim N(\mathbf{0}, c^2 \cdot \mathbf{I}_k)} \frac{n}{\sum_{i=1}^n ||a_i||_2} \tag{64}$$

By central limit theorem, we have $\sqrt{n}(\frac{1}{n}\sum_{i=1}^{n}||a_i||_2-\mu)\sim N(\mathbf{0},\sigma^2)$. And let $g(x)=\frac{1}{x}$, we can use Delta method[9] to find the distribution of $g(\frac{1}{n}\sum_{i=1}^{n}||a_i||_2)$:

$$\sqrt{n}\left(g(\frac{\sum_{i=1}^{n}||a_i||_2}{n})-g(\mu))\right)\sim N(0,\sigma^2\cdot[g\prime(\mu)]^2)=N(0,\sigma^2\cdot\frac{1}{\mu^4}). \tag{65}$$

where μ and σ^2 denote the mean and variance of $||a_i||_2$ respectively. From Eq. (64), we have

$$\begin{aligned}
\mathbf{Var}_{a_i\sim N(\mathbf{0},c^2\cdot\mathbf{I}_k)}\frac{\sum_{i=1}^{n}||a_i||_2}{\sum_{i=1}^{n}||a_i||_2^2} &\approx \mathbf{Var}_{a_i\sim N(\mathbf{0},c^2\cdot\mathbf{I}_k)}\frac{n}{\sum_{i=1}^{n}||a_i||_2} \\
&=\sigma^2\cdot\frac{1}{\mu^4\cdot n} && \text{Since Eq. (65)} \\
&=2c^2\left[\frac{\Gamma(\frac{k}{2}+1)}{\Gamma(\frac{k}{2})}-\frac{\Gamma(\frac{k+1}{2})^2}{\Gamma(\frac{k}{2})^2}\right]\cdot\frac{1}{(\sqrt{2}c\cdot\frac{\Gamma(\frac{k+1}{2})}{\Gamma(\frac{k}{2})})^4\cdot n} \\
& && \text{Since Eq. (12) and Eq. (13)} \\
&=\frac{1}{2c^2\cdot nk^2} && \text{Since Lemma. 2}
\end{aligned}$$

Since Eq. (62), we have

$$\mathbf{Var}_{a_i}\frac{F_1(a_i)}{F_2(a_i)}=\left(\frac{nc^2k}{nc\sqrt{k}}\right)^2\cdot\mathbf{Var}_{a_i}\left(\frac{\sum_{i=1}^{n}||a_i||_2}{\sum_{i=1}^{n}||a_i||_2^2}\right)\approx\frac{1}{2nk}. \tag{66}$$

Similar to Eq. (64),

$$\mathbf{Var}_{a_i\sim N(\mathbf{0},c^2\cdot\mathbf{I}_k)}\frac{\sum_{i=1}^{n}||a_i||_2^2}{\sum_{i=1}^{n}||a_i||_2}\approx\mathbf{Var}_{a_i\sim N(\mathbf{0},c^2\cdot\mathbf{I}_k)}\frac{\sum_{i=1}^{n}||a_i||_2}{n} \tag{67}$$

$$\begin{aligned}
\mathbf{Var}_{a_i\sim N(\mathbf{0},c^2\cdot\mathbf{I}_k)}\frac{\sum_{i=1}^{n}||a_i||_2^2}{\sum_{i=1}^{n}||a_i||_2} &\approx \mathbf{Var}_{a_i\sim N(\mathbf{0},c^2\cdot\mathbf{I}_k)}\frac{\sum_{i=1}^{n}||a_i||_2}{n} && \text{Similar to Eq. (64)} \\
&=\sigma^2\cdot\frac{1}{n} && \text{Since central limit theorem} \\
&=2c^2\left[\frac{\Gamma(\frac{k}{2}+1)}{\Gamma(\frac{k}{2})}-\frac{\Gamma(\frac{k+1}{2})^2}{\Gamma(\frac{k}{2})^2}\right]\cdot\frac{1}{n} && \text{Since Eq. (13)} \\
&=\frac{c^2}{2n} && \text{Since Lemma. 2}
\end{aligned}$$

Since Eq. (63), we have

$$\mathbf{Var}_{a_i}\frac{F_2(a_i)}{F_1(a_i)}=\left(\frac{nc\sqrt{k}}{nc^2k}\right)^2\cdot\mathbf{Var}_{a_i}\left(\frac{\sum_{i=1}^{n}||a_i||_2^2}{\sum_{i=1}^{n}||a_i||_2}\right)\approx\frac{1}{2nk}. \tag{68}$$

From Eq.(66) and Eq.(68), **Theorem 4** holds.

□

In Fig. 11, we also show a numerical verification of **Theorem 4**.

[9] `https://en.wikipedia.org/wiki/Delta_method`

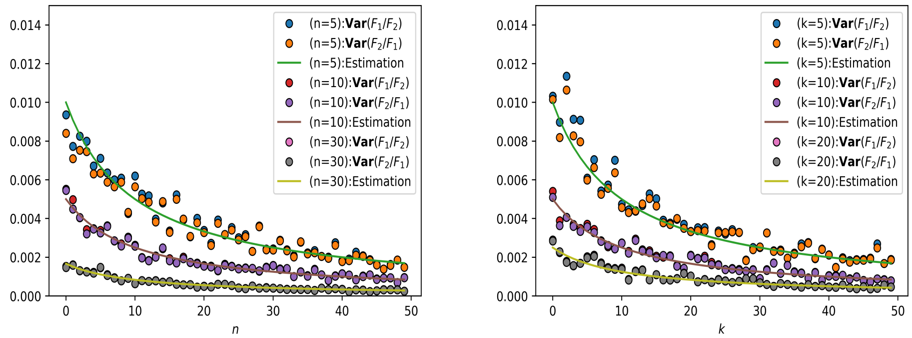


Figure 11: A numerical verification of **Theorem 4**, where $F_1 = \sum_{i=1}^{n} ||a_i||_2 / \mathbb{E}(\sum_{i=1}^{n} ||a_i||_2)$ and $F_2 = \sum_{i=1}^{n} ||a_i||_2^2 / \mathbb{E}(\sum_{i=1}^{n} ||a_i||_2^2)$. a_i follow $N(\mathbf{0}, 0.01^2 \cdot I_k)$.

G Proof of Theorem 5

Proposition 14. *For a $n \times m$ random matrix $(a_{ij})_{n \times m}$, where $a_{ij} \sim N(0, \sigma^2)$. And Eq. (14) holds with probability 1.*

$$\mathbf{rank}((a_{ij})_{n \times m}) = \mathbf{min}(m, n). \tag{69}$$

Lemma 6. *Let $v_0, v_1, ..., v_k$ be the $k+1$ vectors in n dimensional Euclidean space V and $k \leq n$. If $\mathbf{rank}(v_1 - v_0, v_2 - v_0, ..., v_k - v_0) = n$, then $\forall x \in V$, $\exists \lambda_i (0 \leq i \leq k)$, s.t.*

$$x = \sum_{i=0}^{k} \lambda_i \cdot v_i, \tag{70}$$

and $\sum_{i=0}^{k} \lambda_i = 1$. We call $\lambda = (\lambda_0, \lambda_1, ..., \lambda_k)$ the generalized barycentric coordinate with respect to $(v_0, v_1, ..., v_k)$. (In general, barycentric coordinate is a concept in Polytope)

Proof. Note that v_i is the element of n dimensional linear space V and $\mathbf{rank}(v_1 - v_0, v_2 - v_0, ..., v_k - v_0) = n$. It means $(v_1 - v_0, v_2 - v_0, ..., v_k - v_0)$ form a set of basis in the linear space V. $\forall x \in V$, $x - v_0$ can be expressed linearly by them, *i.e.*,$\exists t_i (1 \leq i \leq k)$ s.t.

$$\begin{aligned} x &= v_0 + \sum_{i=1}^{k} t_i (v_i - v_0) \\ &= (1 - \sum_{i=1}^{k} t_i) v_0 + \sum_{i=1}^{k} t_i v_i. \end{aligned}$$

Let $\lambda_0 = (1 - \sum_{i=1}^{k} t_i)$ and $\lambda_i = t_i (1 \leq i \leq k)$, Lemma 6 holds. □

Lemma 7. *Let $v_0, v_1, ..., v_k$ be the $k+1$ vectors in n dimensional Euclidean space V. $\forall a, b \in V$, and the generalized barycentric coordinate of a, b with respect to $(v_0, v_1, ..., v_k)$ are $\lambda = (\lambda_0, \lambda_1, ..., \lambda_k)^T$ and $\mu = (\mu_0, \mu_1, ..., \mu_k)^T$,respectively. Then*

$$||a - b||_2^2 = (\lambda - \mu)^T D (\lambda - \mu), \tag{71}$$

where $D = (-\frac{1}{2} d_{ij})_{(k+1) \times (k+1)}$, and $d_{ij} = ||v_i - v_j||_2^2$.

Proof. Since Lemma 6, let $R = [v_0, v_1, ..., v_k]_{n \times (k+1)}$, and we have $a = R\lambda$ and $b = R\mu$. Moreover,

$$\begin{aligned} ||a - b||_2^2 &= (a - b)^T (a - b) & (72) \\ &= [R(\lambda - \mu)]^T [R(\lambda - \mu)] & (73) \\ &= (\lambda - \mu)^T R^T R (\lambda - \mu). & (74) \end{aligned}$$

Note that, for $D = (-\frac{1}{2}d_{ij})_{(k+1)\times(k+1)}$,

$$-\frac{1}{2}d_{ij} = -\frac{1}{2}(v_i - v_j)^T(v_i - v_j) \tag{75}$$

$$= v_i^T v_j - \frac{1}{2}(v_i^T v_i + v_j^T v_j). \tag{76}$$

So we have $D = R^T R - \frac{1}{2}\left((v_i^T v_i + v_j^T v_j)_{(k+1)\times(k+1)}\right)$. It can be further simplified to $D = R^T R - \frac{1}{2}(V\alpha^T + \alpha V^T)$, where $V = (v_0^T v_0, ..., v_k^T v_k)^T$ and $\alpha = (1, ..., 1)^T$. So

$$||a - b||_2^2 = (\lambda - \mu)^T R^T R(\lambda - \mu) \tag{77}$$

$$= (\lambda - \mu)^T (D + \frac{1}{2}(V\alpha^T + \alpha V^T))(\lambda - \mu) \tag{78}$$

$$= (\lambda - \mu)^T D(\lambda - \mu) + \frac{1}{2}(\lambda - \mu)^T (V\alpha^T + \alpha V^T)(\lambda - \mu), \tag{79}$$

therefore, we only need to prove $(\lambda - \mu)^T (V\alpha^T + \alpha V^T)(\lambda - \mu) = 0$. From Lemma 6, we have $\alpha^T(\lambda - \mu) = (\lambda - \mu)^T\alpha = 0$ and the Lemma 7 holds.

□

Definition 1 (Ultra dimension)**.** *For a set U composed of vectors in a n dimensional linear space V, we define $\widehat{\mathbf{dim}}(U)$ as the Ultra dimension of U. The definition is that if U has k linearly independent vectors and there are no more, then $\widehat{\mathbf{dim}}(U) = k$.*

In fact, if U is a linear subspace in V, then the Ultra dimension and the dimensions of the linear subspace are equivalent. If U is a linear manifold, $U = \{x + v_0 | x \in W\}$, where v_0 and W are non-zero vectors and linear subspaces in V, respectively. And $\mathbf{dim}(W) = r$. Then

$$\widehat{\mathbf{dim}}(U) = \begin{cases} r, & v_0 \in W \\ r+1, & v_0 \notin W \end{cases} \tag{80}$$

In other words, $\widehat{\mathbf{dim}}(U) \geq \widehat{\mathbf{dim}}(W)$ always holds.

Lemma 8. *For arbitrary k $(1 \leq k \leq n-1)$, let $a_1, a_2, ..., a_k$ be k linearly independent vectors in n dimensional linear space V. Consider one $n-1$ dimensional linear subspace W in V and a non-zero vector v_0 in V. They form a linear manifold $P = \{v_0 + \alpha | \alpha \in W\}$. If $a_1, a_2, ..., a_k$ do not all belong to P, then there must exist $n-k$ vectors $p_1, p_2, ..., p_{n-k}$ from P, s.t $(a_1, a_2, ..., a_k, p_1, p_2, ..., p_{n-k})$ are a set of basis for the linear space V.*

Proof. we use mathematical induction. First, show that the Lemma 8 holds for $n - k = 1$. it means we need to find a vector $p_1 \in P$ s.t. $a_1, a_2, ..., a_k, p_1$ linearly independent. If p_1 does not exist, then $\forall p \in P$ would be linearly represented by $a_1, a_2, ..., a_k$. In other word,

$$P \subset L = \mathbf{span}(a_1, a_2, ..., a_k), \tag{81}$$

① For the linear manifold P, if $v_0 \in W$. This means that P is equal to the linear subspace W. Since Eq. (81), we have $W \subset L$ and $\widehat{\mathbf{dim}}(W) = \widehat{\mathbf{dim}}(L)$. Hence, $P = W = L$. However, $a_1, a_2, ..., a_k$ do not all belong to P, a contradiction.

② For the linear manifold P, if $v_0 \notin W$, then $\widehat{\mathbf{dim}}(P) = n$. Because $v_0 \notin W$, that is, v_0 cannot be represented by a set of basis of W. In other words, v_0 and a set of basis of W are linearly independent. However, the dimension of W is $n - 1$, hence $\widehat{\mathbf{dim}}(P) = n$. From Eq. (81), we have $P \subset L$, so

$$n = \widehat{\mathbf{dim}}(P) \leq \widehat{\mathbf{dim}}(L) = k = n - 1, \tag{82}$$

a contradiction. Therefore, Lemma 8 holds for $n - k = 1$. Assume the induction hypothesis that Lemma 8 is true when $n - k = l$, where $1 \leq l$. when $n - k = l + 1$, *i.e.*, $k = n - (l + 1)$, we also can find a vector $p_1 \in P$ s.t. $a_1, a_2, ..., a_k, p_1$ linearly independent. Otherwise, $\forall p \in P$ would be linearly represented by $a_1, a_2, ..., a_k$. Similarly, we have Eq. (81). Note that, from Definition 1, $\widehat{\mathbf{dim}}(P) \geq n - 1$, hence

$$n - 1 \leq \widehat{\mathbf{dim}}(P) \leq \widehat{\mathbf{dim}}(L) = k = n - (l + 1). \tag{83}$$

a contradiction. At this time, we have $k+1 = n-(l+1)+1 = n-l$ vectors $a_1, a_2, ..., a_k, p_1$ which are not all on P. Note that $n-(n-l) = l$, using the induction hypothesis, the Lemma 8 also holds for $n-k = l$. In summary, Lemma 8 holds.

□

Theorem 5. Let $v_0, v_1, ..., v_k$ be the $k+1$ vectors in n dimensional Euclidean space $\mathbb{E}^n$. For all P in $\mathbb{E}^n$,

$$\sum_{i=0}^{k} ||P - v_i||_2^2 = \sum_{i=0}^{k} ||G - v_i||_2^2 + (k+1)||P - G||_2^2.$$

where G is the centroid of v_i, will hold if it satisfies one of the following conditions:

(1)if $k \geq n$ and $\mathbf{rank}(v_1 - v_0, v_2 - v_0, ..., v_k - v_0) = n$.

(2)if $k < n$ and $(v_1 - v_0, v_2 - v_0, ..., v_k - v_0)$ are linearly independent.

(3)if $v_i \sim N(\mathbf{0}, c \cdot \mathbf{I}_n)$, Eq.(50) holds with probability 1 where c is a constant.

Proof. **For Theorem 5 (1)**. From Lemma 6, $\forall P \in E^n$,$\exists \gamma = (\gamma_0, ..., \gamma_k)$, s.t. P can be represented by $\sum_{i=0}^{k} \gamma_i v_i$, where $\sum_{i=0}^{k} \gamma_i = 1$. In fact, for each v_i, it also can be respresented by $\sum_{j=0}^{k} \beta_{ij} v_i$, where $\sum_{i=0}^{k} \beta_{ij} = 1$. We just take $(\beta_{i0}, \beta_{i1}, ..., \beta_{ik})$ as one of the standard orthogonal basis $\epsilon_i = (0, 0, ..., 1_i, ...0)$. According to lemma 7,

$$\begin{aligned} ||P - v_i||_2^2 &= (\gamma - \epsilon_i)^T D(\gamma - \epsilon_i) && (84)\\ &= \gamma^T D\gamma - 2\gamma^T D\epsilon_i + \epsilon_i^T D\epsilon_i && (85)\\ &= \gamma^T D\gamma - 2\gamma^T D\epsilon_i. && (86) \end{aligned}$$

The final equation is because the diagonal elements of the matrix are all 0. On the other hand, we have

$$\begin{aligned} ||G - v_i||_2^2 &= (\frac{1}{k+1}\sum_{i=0}^{k} \epsilon_i - \epsilon_i)^T D(\frac{1}{k+1}\sum_{i=0}^{k} \epsilon_i - \epsilon_i) && (87)\\ &= \frac{1}{(k+1)^2}\alpha^T D\alpha - \frac{2}{k+1}\alpha^T D\epsilon_i + \epsilon_i^T D\epsilon_i && (88)\\ &= \frac{1}{(k+1)^2}\alpha^T D\alpha - \frac{2}{k+1}\alpha^T D\epsilon_i, && (89) \end{aligned}$$

where $\alpha = \sum_{i=0}^{k} \epsilon_i$, *i.e.*,$\alpha = (1, 1, ..., 1)$. Next, we consider $||P - G||_2^2$.

$$\begin{aligned} ||P - G||_2^2 &= (\gamma - \frac{1}{k+1}\alpha)^T D(\gamma - \frac{1}{k+1}\alpha) && (90)\\ &= \gamma^T D\gamma + \frac{1}{(k+1)^2}\alpha^T D\alpha - \frac{2}{k+1}\gamma^T D\alpha. && (91) \end{aligned}$$

In summary, we have

$$\begin{aligned} \sum_{i=0}^{k} ||P - v_i||_2^2 - ||G - v_i||_2^2 &= (k+1)\gamma^T D\gamma - 2\gamma^T D\alpha + \frac{1}{k+1}\alpha^T D\alpha && (92)\\ &= (k+1)||P - G||_2^2 && (93) \end{aligned}$$

Therefore, Theorem 5 (1) holds.

For Theorem 5 (2). Next, we prove the case of $k < n$. Obviously, Lemma 6 does not hold. We consider about such a linear space $W_1 = \mathbf{span}(P - G)$, *i.e.*, a linear space expanded by $P - G$, and its orthogonal complement $W_1^{\perp}$ (in E^n). Since dimension formula from linear space, it is easy to konw that $\mathbf{dim}(W_1^{\perp}) = n - 1$.

Two linear manifolds T_1 and T_2 are constructed as follows,

$$T_1 = \{x + G | x \in W_1^{\perp}\} \tag{94}$$

$$T_2 = \{x + G - v_0 | x \in W_1^{\perp}\} \tag{95}$$

$\forall v_i \in T_1$, we have $(v_i - G)^T(P - G) = 0$, Furthermore,

$$||P - v_i||_2^2 = ||v_i - G||_2^2 + ||P - G||_2^2. \tag{96}$$

It is easy to know that $G - v_0$ is not 0. If $v_1 - v_0, ..., v_k - v_0$ are all belong to T_2, it means $v_1, .., v_k$ are all in T_1. Hence, we have Eq. (96). By summing both sides of Eq. (96) for i, it is obvious find that Theorem 5 (2) holds. If $v_1 - v_0, ..., v_k - v_0$ are not all belong to T_2, since Lemma 8, there are $n-k$ vectors $p_1 - v_0, p_2 - v_0, .., p_{n-k} - v_0$ from T_2 s.t. they and $v_1 - v_0, ..., v_k - v_0$ are linearly independent, where p_i obviously belongs to manifold T_1.

At the same time, we have $2G - p_i \in T_1$, we can also construct $n - k$ new vectors $2G - p_i - v_0 \in T_2$ and calculate the rank that

$$\begin{aligned}
&\mathbf{rank}(v_1 - v_0, ..., v_k - v_0, p_1 - v_0, ..., p_{n-k} - v_0, 2G - p_1 - v_0, ..., 2G - p_{n-k} - v_0) \\
&= \mathbf{rank}(v_1 - v_0, ..., v_k - v_0, p_1 - v_0, ..., p_{n-k} - v_0, 2(G - v_0), ..., 2(G - v_0)) &(97)\\
&= \mathbf{rank}(v_1 - v_0, ..., v_k - v_0, p_1 - v_0, ..., p_{n-k} - v_0, 0, ..., 0) &(98)\\
&= n &(99)
\end{aligned}$$

The reason of the final equation is that $\sum_{i=1}^{k}(v_i - v_0) = (k + 1)(G - v_0)$. Note that there are a total of $k + (n - k) + (n - k) = n + (n - k) \geq n$ vectors, meets the lemma 6 condition. For the convenience of description, we define

$$L_i^{(1)} = v_i, (0 \leq i \leq k), \tag{100}$$

$$L_i^{(2)} = p_i, (1 \leq i \leq n - k), \tag{101}$$

$$L_i^{(3)} = 2G - p_i, (1 \leq i \leq n - k). \tag{102}$$

And their centroid is

$$\begin{aligned}
G' &= \frac{1}{2n - k + 1}\left(\sum_{i=0}^{k} v_i + \sum_{i=1}^{n-k}(L_i^{(2)} + L_i^{(3)})\right) &(103)\\
&= \frac{1}{2n - k + 1}((k + 1)G + 2(n - k)G) &(104)\\
&= G &(105)
\end{aligned}$$

That is, the newly added vector does not change the centroid of v_i. On the other hand, since both $L_i^{(2)}$ and $L_i^{(3)}$ are in the linear manifold T_1, and it meets the conditions of the Eq.(96). Similar to the derivation in the Theorem 5 (1), we have

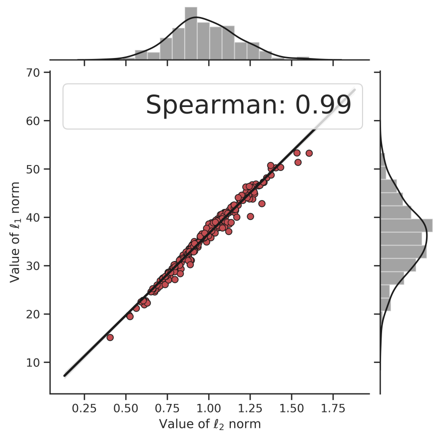
$$\begin{aligned}
(2n - k + 1)||P - G||_2^2 &= \sum_{t = L_i^{(1)}, L_i^{(2)}, L_i^{(3)}} \left(||P - t||_2^2 - ||G - t||_2^2\right) &(106)\\
&= \sum_{i=0}^{k}\left(||P - v_i||_2^2 - ||G - v_i||_2^2\right) + \sum_{t = L_i^{(2)}, L_i^{(3)}}\left(||P - t||_2^2 - ||G - t||_2^2\right) &(107)\\
&= \sum_{i=0}^{k}\left(||P - v_i||_2^2 - ||G - v_i||_2^2\right) + 2(n - k)||P - G||_2^2 &(108)
\end{aligned}$$

The final equation is because both $L_i^{(2)}$ and $L_i^{(3)}$ are in the linear manifold T_1 and satisfy Eq. (96). To simplify Eq. (108), we obtain $\sum_{i=0}^{k}\left(||P - v_i||_2^2 - ||G - v_i||_2^2\right) = (k + 1)||P - G||_2^2$. Therefore, Theorem 5 (2) holds.

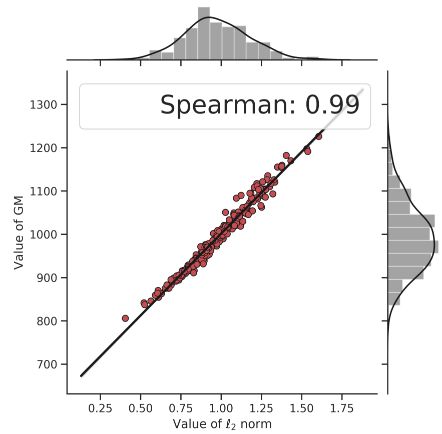
For Theorem 5 (3). When $k \geq n$, from Proposition 14, we know that $\mathbf{rank}(v_1 - v_0, v_2 - v_0, ..., v_k - v_0)$ = n holds with probability 1. Hence, if we use the similar deduction from Theorem 5 (1), we can find that Theorem 5 (3) holds when $k \geq n$. On the other hand, when $k < n$, we can get the same result also according to Proposition 14. The reason is that $(v_1 - v_0, v_2 - v_0, ..., v_k - v_0)$ are linearly independent with probability 1.

□

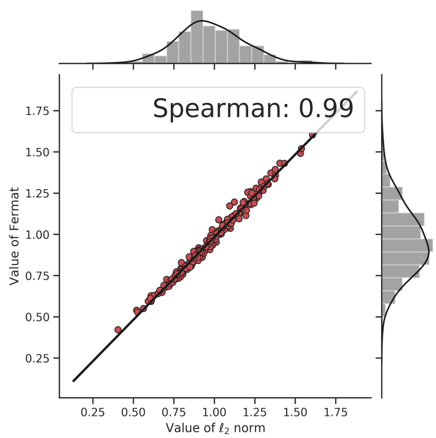
H The result of Sp



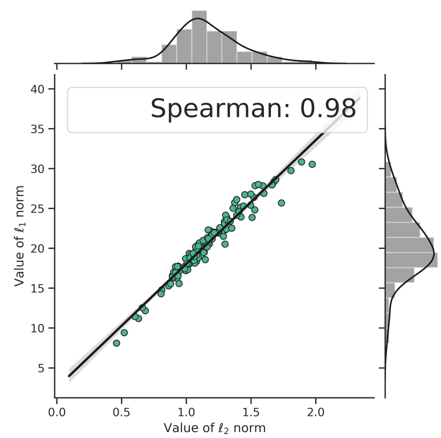
(a) Sp = 0.99



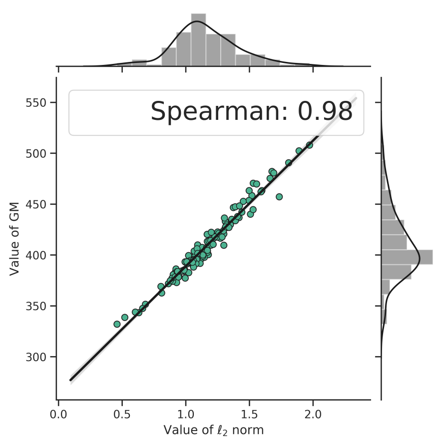
(b) Sp = 0.99



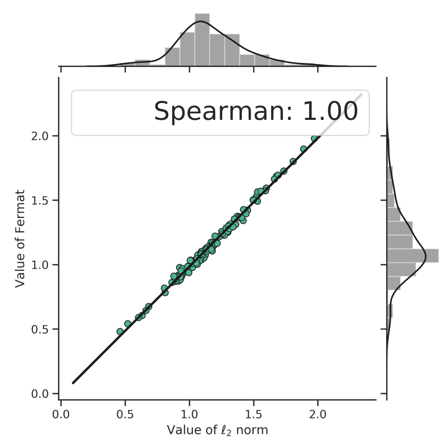
(c) Sp = 0.99



(d) Sp = 0.98



(e) Sp = 0.98



(f) Sp = 1.00

Figure 12: The Spearman's rank correlation coefficient (Sp) for different criteria. (a-c) are Sp between ℓ_1 and ℓ_2, $\mathbf{GM}$ and ℓ_2, $\mathbf{Fermat}$ and ℓ_2 from ResNet18 (12^{th} Conv), respectively. The results of VGG16 (3^{rd} Conv) are shown in (d-f). If the Sp of two pruning criteria is close to 1, then the sequence of their pruned filters may have strong similarity.

I Other result

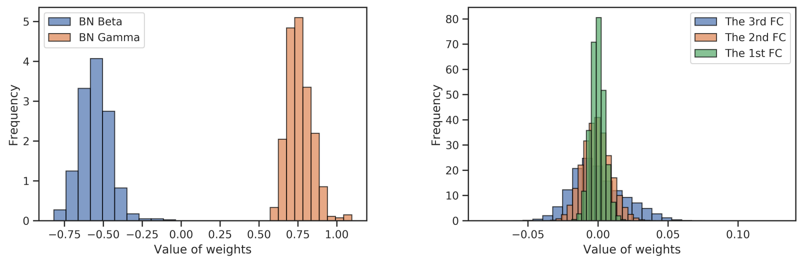


Figure 13: The distribution about other learnable parameters. (Left): The disrtibution about the learnable parameters of batch normalization. (Rihgt): The parameters distribution of the fully-connected layers (FC). For FC, the Sp between the criteria in Table2 are greater than 0.9.

In Fig 13, we show the other learnable parameters (*i.e.* Batch normalization (BN) and fully connected neural network (FC)) in VGG16-BN. For BN, the distribution of its parameters does not satisfy CWDA, and similar results are shown in [34, 35]. Moreover, the learnable parameters of fully-connected layers also do not follow a Gaussian-alike distribution, which is consistent with the conclusion in previous work [36, 37, 38].

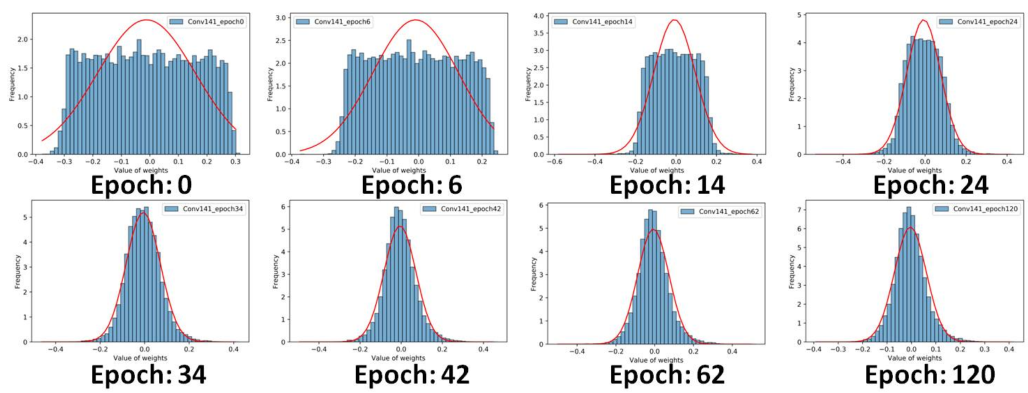


Figure 14: The distribution of the convolutional filter (141^{th} Conv) with kaiming-uniform initialization for each epoch.

J An interesting case for *Importance Score* measured by different criteria

The following results are the index of pruned filters obtained by the filters' *Importance Score* from different types of pruning criteria. We take VGG16 (2^{nd}) as an example. The 5^{th} filter in this layer is regarded as a redundant convolutional filter for APoZ criterion, but other criteria consider it to be almost the most important.

Taylor ℓ_1: [27, 36, 25, 11, 6, 23, 24, 16, 0, 57, 48, 53, 1, 61, 18, 55, 34, 15, 51, 58, 31, 3, 12, 21, 59, 30, 7, 38, 41, 50, 10, 33, 17, 46, 62, 13, 49, 43, 42, 47, 2, 32, 44, 20, 39, 52, 56, 40, 9, 26, 37, 22, 29, 54, 60, 8, 14, 45, 4, 63, 19, 35, 28, **5**]

Taylor ℓ_2: [23, 32, 36, 11, 62, 16, 30, 59, 10, 13, 2, 50, 38, 0, 46, 43, 21, 26, 15, 22, 7, 51, 39, 33, 14, 58, 9, 40, 57, 6, 61, 44, 20, 48, 3, 53, 41, 56, 17, 12, 18, 31, 4, 1, 25, 19, 63, 24, 54, 45, 52, 37, 55, 47, 34, 35, 8, 29, 42, 27, 49, 28, 60, **5**]

BN_β: [52, 46, 32, 21, 14, 29, 17, 0, 19, 36, 1, 51, 44, 40, 41, 60, 57, 27, 22, 53, 63, 8, 30, 26, 23, 58, 39, 18, 9, 47, 31, 35, 11, 37, 55, 45, 3, 61, 6, 4, 33, 25, 15, 48, 43, 28, 56, 2, 13, 16, 34, 20, 59, 10, 7, 24, 50, 62, 12, 49, 38, 42, **5**, 54]

APoZ: [**5**, 10, 38, 42, 62, 24, 13, 12, 7, 28, 59, 15, 23, 11, 16, 56, 34, 35, 57, 19, 2, 49, 43, 25, 6, 63, 61, 36, 9, 27, 33, 20, 48, 58, 55, 18, 51, 31, 1, 0, 53, 37, 26, 29, 47, 60, 8, 44, 41, 46, 21, 17, 14, 32, 52, 22, 39, 3, 40, 30, 4, 45, 50, 54]

K The details of other pruning criteria

For notation, we denote i^{th} convolutional filter in layer l as F_i^l and the input feature maps in layer l as $\mathbf{I}^l \in \mathbb{R}^{N\times I^l\times H^l\times W^l}$, where N, I^l, H^l, W_l mean the train set size, number of channels, height and width respectively, $i = 1, 2, \cdots, \lambda_l$, and $l = 1, 2, \cdots, L$. The formulation of the filters' *Importance Score* under each pruning criteria are illustrated as follows:

Norm-based criteria:

- ℓ_1-Norm [5]: $||F_i^l||_1$;
- ℓ_2-Norm [7]: $||F_i^l||_2$;

BN-based criteria [12]:

- BN_γ: $|\gamma_i^l|$, where γ_i^l is the scaling factor in the Batch Normalization layer l;
- BN_β: $|\beta_i^l|$, where β_i^l is the shifting factor in the Batch Normalization layer l.

Activation-based criteria:

- APoZ [8]: $\frac{\sum_{p,q}\mathbb{1}\left((|\mathbf{I}^l * F_i^l|)_{p,q}>\sigma\right)}{N\times I^l\times H^l\times W^l}$, where we set $\sigma = 0.0001$ same as [9], and $\mathbb{1}(\cdot)$ is the indicator function, $*$ is convolution operator and $\mathbf{I}^l * F_i^l$ is the i-th output feature map;
- Entropy [9]: we first prepare $\mathbf{G}_i^l = GAP(\mathbf{I}^l * F_i^l)$, where $\mathbf{G}_i^l \in \mathbb{R}^{N\times 1}$ and $GAP(\cdot)$ is the Global Average Pooling. Then, we estimate statistical distribution for $\mathbf{G}_i^l$ by dividing all elements in $\mathbf{G}_i^l$ into m bins. Let p_j is the probability of bin j, and the the *Importance Score* score is $-\sum_{j=1}^{m} p_j \log p_j$.

First order Taylor based criteria [10, 11, 26]:

- Taylor ℓ_1-Norm: $||\frac{\partial loss}{\partial F_i^l} \cdot F_i^l||_1$;
- Taylor ℓ_2-Norm: $||\frac{\partial loss}{\partial F_i^l} \cdot F_i^l||_2$;

The $loss$ is the Cross Entropy Loss on the split training set from the original training set.

L Additional experiments about image clasification

Table 5: The accuracy(%) of several networks and datasets using different pruning criteria.

		Experiment (1)			Experiment (2)			Experiment (3)		
		Trained	Pruned	Fine-tuned	Trained	Pruned	Fine-tuned	Trained	Pruned	Fine-tuned
CIFAR10 VGG16	ℓ_1	93.61	61.21	93.51	93.21	54.31	93.22	93.26	57.74	93.32
	ℓ_2	93.61	63.41	93.32	93.21	54.61	93.42	93.26	57.42	93.29
	GM	93.61	61.22	93.41	93.21	53.71	93.25	93.26	57.46	93.36
CIFAR100 VGG16	ℓ_1	72.67	25.91	71.50	72.99	20.43	71.36	72.56	24.01	71.07
	ℓ_2	72.67	27.07	71.28	72.99	22.31	71.12	72.56	24.45	70.92
	GM	72.67	26.37	71.27	72.99	21.67	71.26	72.56	24.26	70.78
ImageNet VGG16	ℓ_1	71.58	30.33	71.02	71.33	40.33	70.12	72.01	28.07	70.93
	ℓ_2	71.58	29.47	70.83	71.33	40.45	70.13	72.01	27.89	71.02
	GM	71.58	30.76	70.95	71.33	39.86	70.33	72.01	28.01	70.74
CIFAR10 ResNet56	ℓ_1	92.98	77.73	93.08	92.97	76.02	92.82	93.01	79.93	92.81
	ℓ_2	92.98	79.02	92.83	92.97	77.91	92.72	93.01	82.43	92.81
	GM	92.98	74.26	92.77	93.2	73.93	92.61	93.01	80.48	92.84
CIFAR100 ResNet56	ℓ_1	71.36	50.64	70.15	70.02	52.41	69.19	70.48	52.19	69.77
	ℓ_2	71.36	53.44	70.16	70.02	52.73	69.31	70.48	52.16	69.62
	GM	71.36	45.12	70.22	70.02	52.62	69.54	70.48	50.74	69.69
ImageNet ResNet34	ℓ_1	73.31	62.22	73.06	73.16	54.24	72.99	73.21	63.12	73.02
	ℓ_2	73.31	62.02	72.91	73.16	53.64	72.78	73.21	62.98	72.86
	GM	73.31	61.88	72.96	73.16	53.48	72.94	73.21	62.36	73.04

All the setting of these experiments are under can be found in `https://github.com/bearpaw/pytorch-classification`. Specifically, for pruning ratio:

VGG16 on CIFAR10, CIFAR100 and ImageNet:

`https://github.com/Eric-mingjie/rethinking-network-pruning/blob/master/cifar/l1-norm-pruning/vggprune.py#L84`

ResNet56 on CIFAR10 and CIFAR100:

`https://github.com/Eric-mingjie/rethinking-network-pruning/blob/master/cifar/l1-norm-pruning/res56prune.py#L94`

ResNet34 on ImageNet:

`https://github.com/Eric-mingjie/rethinking-network-pruning/blob/master/imagenet/l1-norm-pruning/prune.py#L138`

M About weight decay

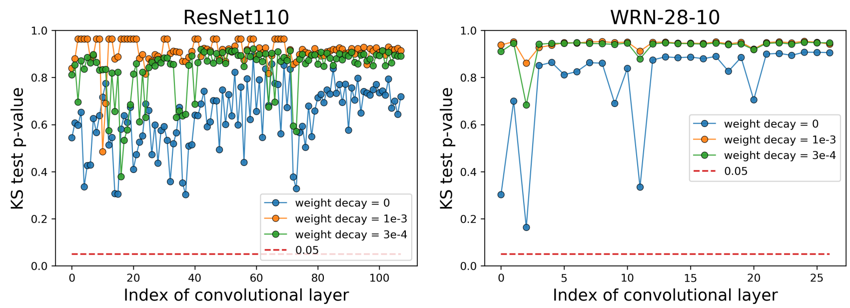


Figure 15: KS test [39] while using different settings of weight decay.

We train the ResNet110 and WRN-28-10 on CIFAR100 with different weight decay (1e-3, 3e-4 and 0) and use KS test to verify whether the parameters of different layers follow a normal distribution. In Fig. 15, we can find

(1) When weight decay (wd) is non-zero, the normality is higher than that when weight decay is 0.

(2) If weight decay is 0, the p-value can still be much greater than 0.05, which means that the regularization of weight decay may not be the key reason for CWDA. The distribution of the parameters in these two networks (weight decay is 0) are shown in Fig. 17 and Fig. 16.

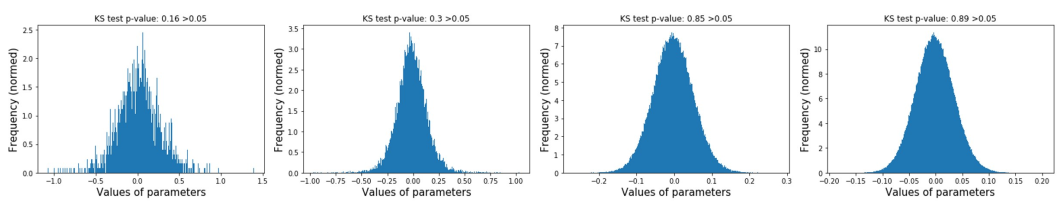


Figure 16: The distribution of parameters in different convolutional filters (WRN-28-10, wd = 0).

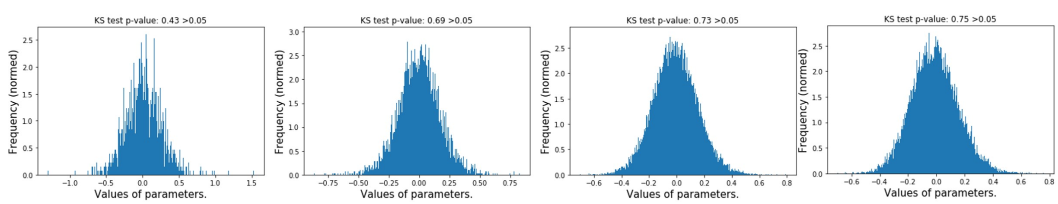
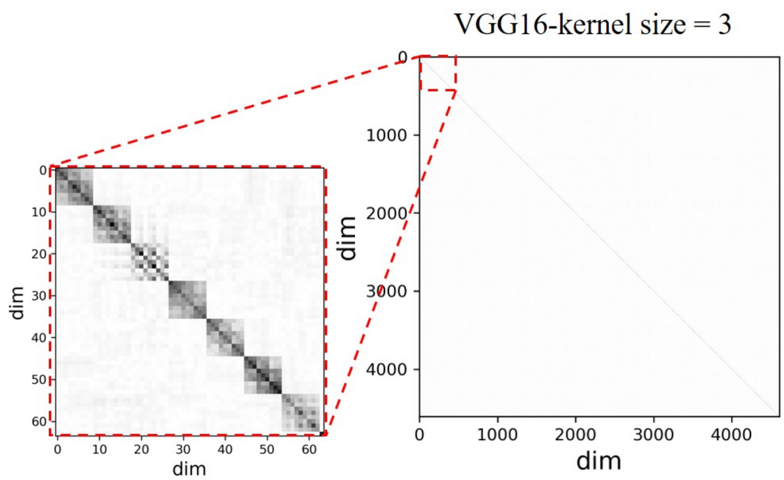
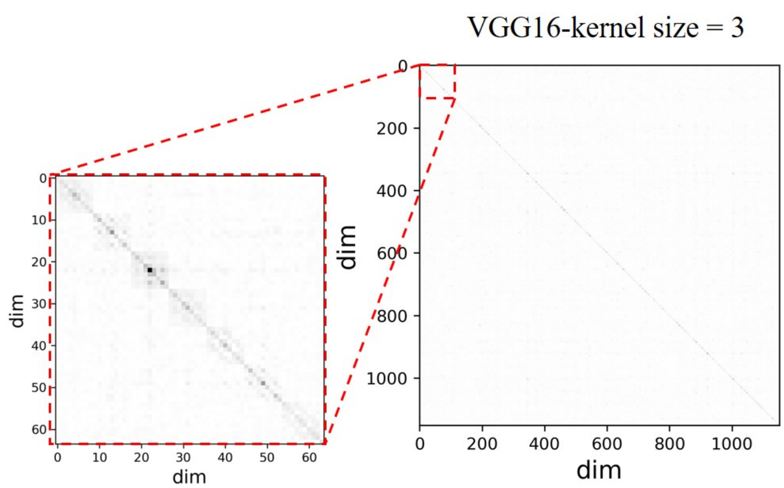


Figure 17: The distribution of parameters in different convolutional filters (ResNet110, wd = 0).

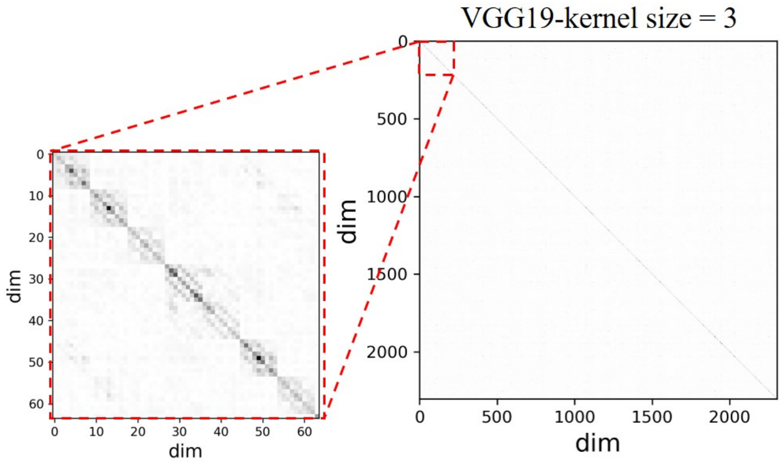
N More visualizations of correlation matrix

N.1 VGG16

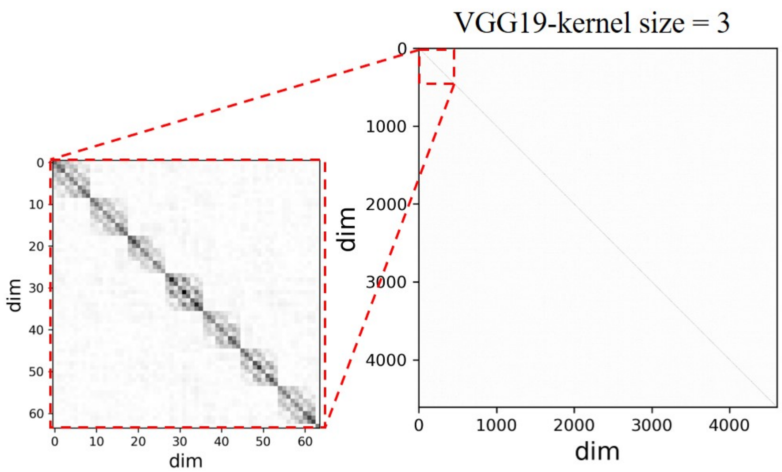




N.2 VGG19

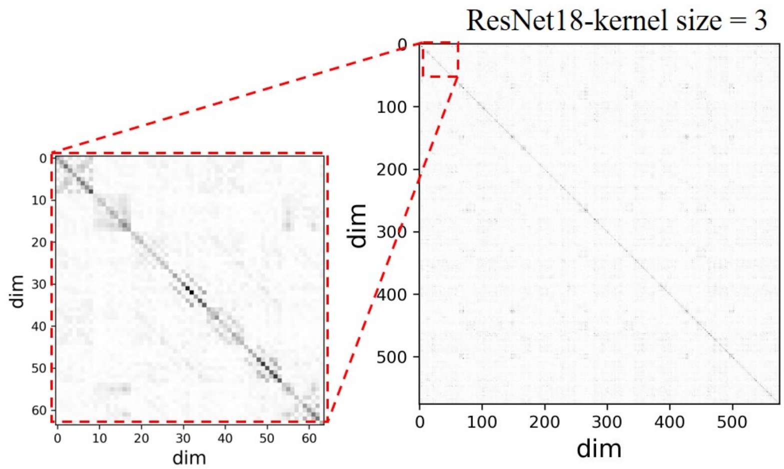


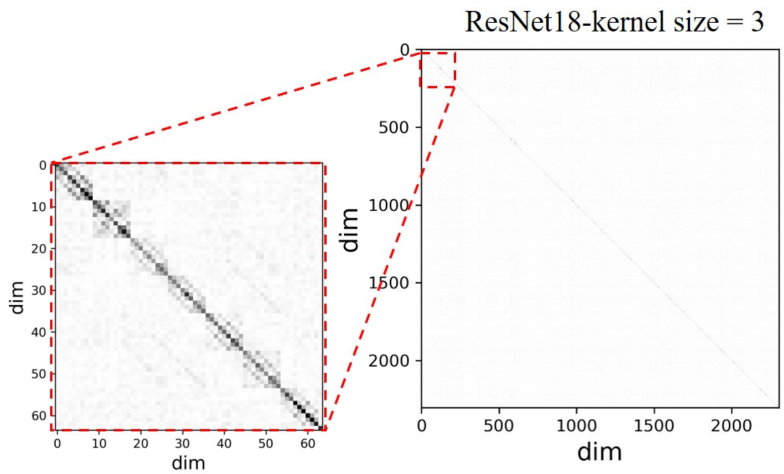
VGG19-kernel size = 3
dim
dim
dim
dim



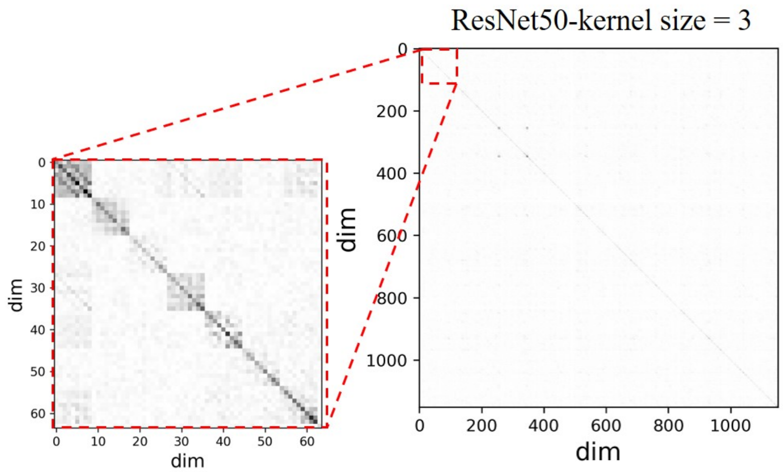
VGG19-kernel size = 3
dim
dim
dim
dim

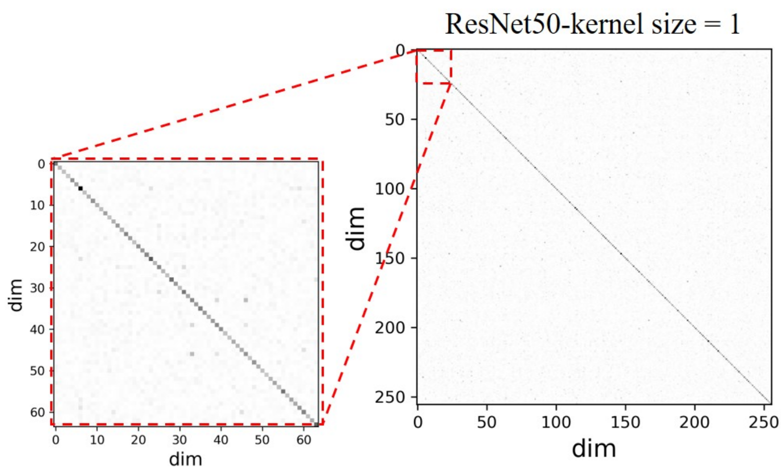
N.3 ResNet18


ResNet18-kernel size = 3
dim
dim
dim
dim

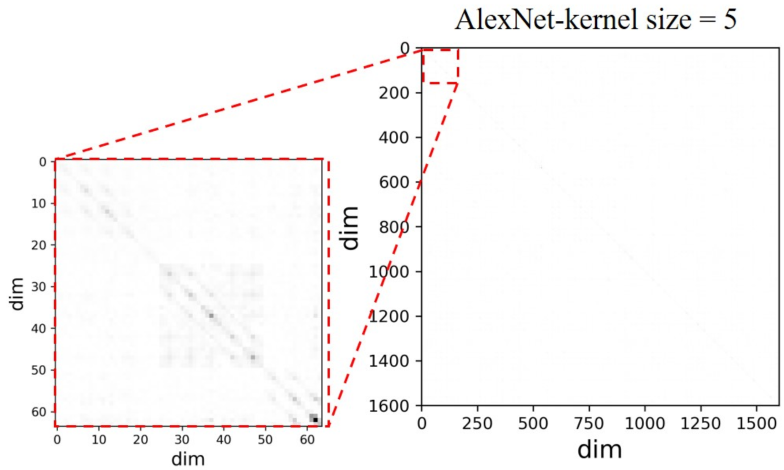

ResNet18-kernel size = 3
dim
dim
dim
dim

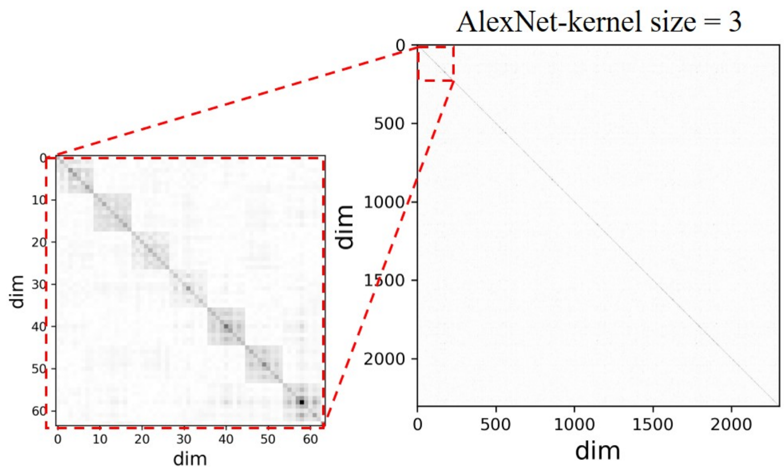
N.4 ResNet50


ResNet50-kernel size = 3
dim
dim
dim
dim

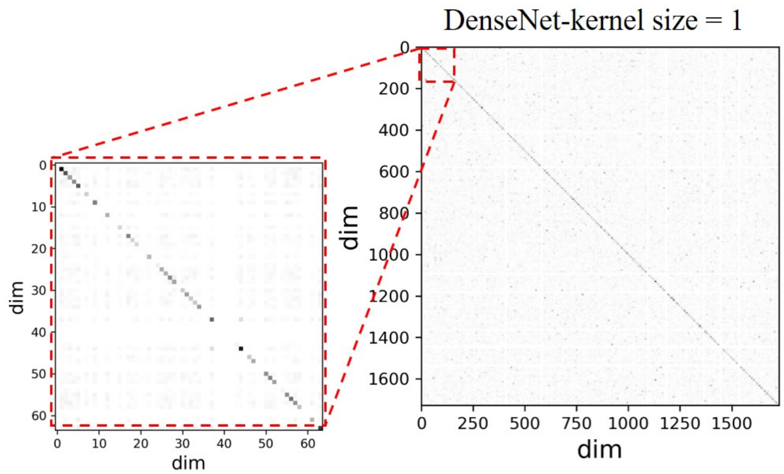

ResNet50-kernel size = 1
dim
dim
dim
dim

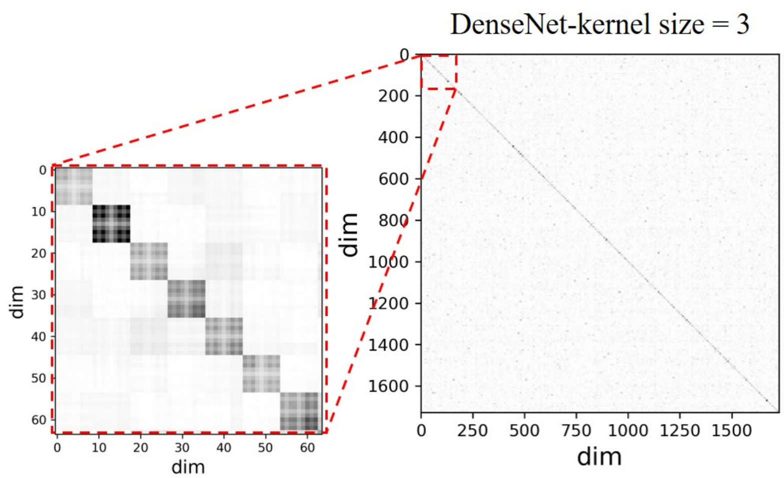
N.5 AlexNet


AlexNet-kernel size = 5
dim
dim
dim
dim

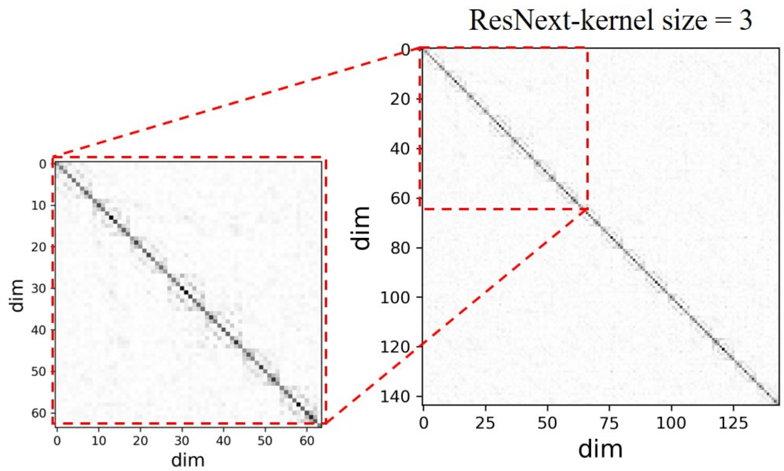

AlexNet-kernel size = 3
dim
dim
dim
dim

N.6 DenseNet

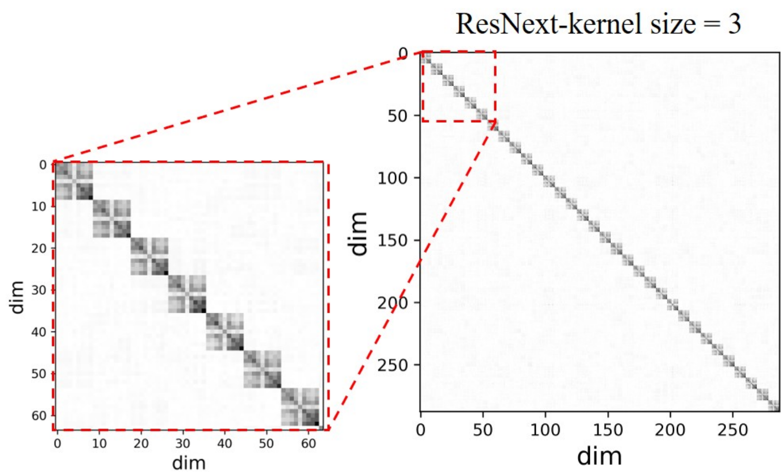

DenseNet-kernel size = 1
dim
dim
0
10
20
30
40
50
60
0
200
400
600
800
1000
1200
1400
1600
250
500
750
1250
1500


DenseNet-kernel size = 3
dim
dim
0
10
20
30
40
50
60
0
200
400
600
800
1000
1200
1400
1600
250
500
750
1250
1500

N.7 ResNext

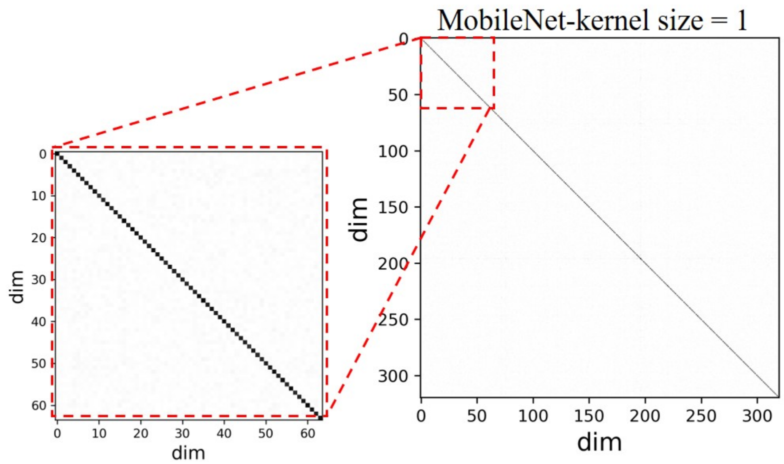


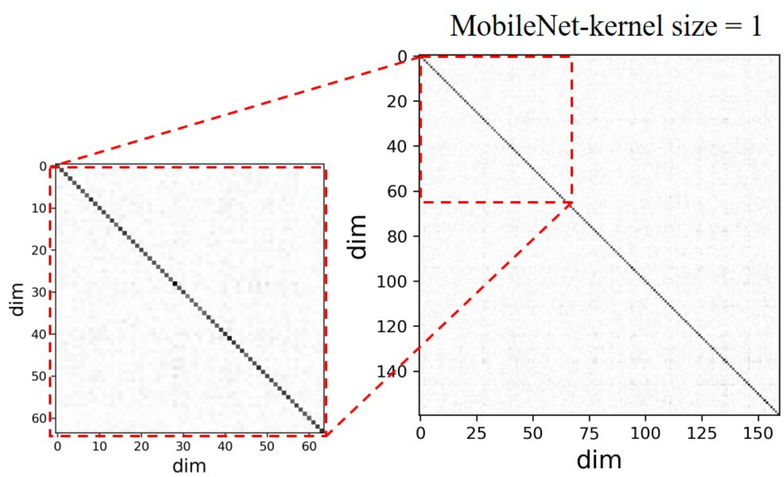
ResNext-kernel size = 3
dim
dim
dim
dim



ResNext-kernel size = 3
dim
dim
dim
dim

N.8 MobileNet





O More experiments for supporting our analysis in global pruning

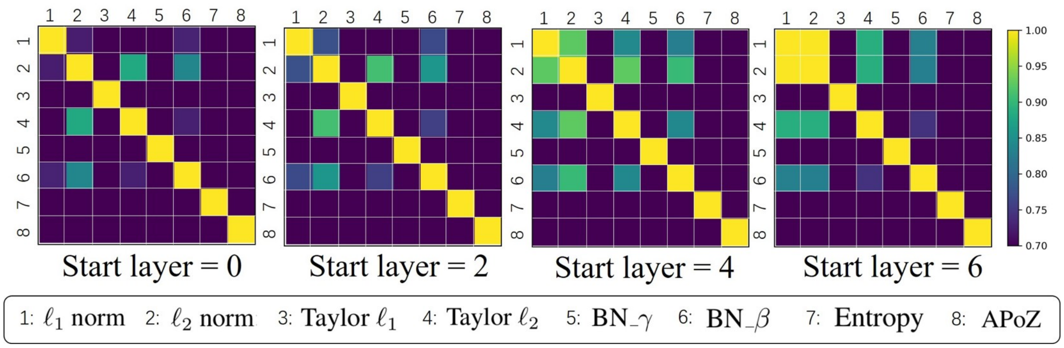


Figure 18: Global pruning with different start layer.

For VGG16. As shown in Fig.6 (a-b), compared with ResNet56, VGG16 has some layers with different dimensions but similar *Importance Score* measured by ℓ_1 or ℓ_2, such as "layer 2" and "layer 8" for ℓ_2 criterion in Fig.6 (a). From Table 3 (3-4), these pairs of layers make the Sp small, which explain why the result of ℓ_1 and ℓ_2 pruning is not similar in Fig. 5 (e) for VGG16. We consider a special class of global pruning, *i.e.,* the convolutional filters from one middle layer (called "Start layer") to the last layer are pruned globally. According to our analysis and Fig.6 (a-b), we can deduce that when "Start layer" ≥ 4, the Sp between ℓ_1 and ℓ_2 is large enough. The experiments in Fig.18 are consistent with our analysis, which imply our analysis is reasonable.

P Statistical Test

In this section, according to Section 2.1, we have a series of statistical tests for the necessary conditions of CWDA. let $F_{ij} \in \mathbb{R}^{N_i \times k \times k}$ represent the j^{th} filter of the i^{th} convolutional layer.[10]

(1) **Gaussian**. We verify whether F_{ij} approximatively follow a Gaussian-alike distribution. In i^{th} layer, we use Kolmogorov–Smirnov (KS) test [39] to check if all the weights in the same layer follow a normal distribution.

(2) **Variance**. We verify whether the variance of the diagonal elements of Σ_{diag} are small enough. Since Appendix B, Let σ_j denotes the standard deviation of all the weights of filter F_{ij} in i^{th} layer. We use Student's t test [40] to check if the variance of these σ_j is small enough. The null hypothesis H_0 and the alternative hypothesis H_1 are:

$$H_0 : \mathbf{Var}(\sigma_1^2, \sigma_2^2, .., \sigma_{N_i}^2) \le \sigma_0^2, \qquad H_1 : \mathbf{Var}(\sigma_1^2, \sigma_2^2, .., \sigma_{N_i}^2) > \sigma_0^2.$$

where N_i denotes the number of the filters in i^{th} layer and σ_0 is a given real number which is small enough, like $\sigma_0^2 = 0.0001$.

(3) **Mean**. We verify whether the mean of F_{ij} is 0. Let the mean of all the weights in the same layer is μ. We use Student's t test [40] to check if μ is close to 0. First, we check the upper bound (Mean-Left) of μ, *i.e.*,

$$H_0 : \mu \le \epsilon_0, \qquad H_1 : \mu > \epsilon_0.$$

where ϵ_0 is a small constant, like $\epsilon_0 = 0.01$. Next, we check the lower bound (Mean-Right) and the null hypothesis H_0 and the alternative hypothesis H_1 are:

$$H_0 : \mu \ge -\epsilon_0, \qquad H_1 : \mu < -\epsilon_0.$$

(4) **Magnitude**. We verify whether ϵ is small enough. Let h denote the mean of the off-diagonal elements of $\mathbf{\Sigma}_{\text{diag}} + \epsilon \cdot \mathbf{\Sigma}_{\text{block}}$.

$$H_0 : h \le \epsilon_0, \qquad H_1 : h > \epsilon_0.$$

Table 6: The experiments for having the comprehensive statistical tests on CWDA.

NETWORK STRUCTURE	**OPTIMIZER**	**REGULARIZATION**
ResNet [41]	SGD [42]	L1 norm
VGG [43]	ASGD [44]	L2 norm
AlexNet [45]	Adam [46]	RReLu [47]
DenseNet [48]	Adagrad [49]	Dropact [50]
PreResNet [51]	Adamax [46]	Autoaug [52]
WRN [53]	Adadelta [54]	Cutout [55]
ResNext [56]		Cutmix [57]
ATTENTION MECHANISM	**INITIALIZATION**	**DATASET**
SENet [58]	Kaiming-normal [59]	CIFAR10 [60]
DIANet [61]	Kaiming-uniform [59]	CIFAR100 [60]
SRMNet [62]	Xavier-normal [63]	ImageNet [64]
CBAM [65]	Xavier-uniform [63]	MNIST [66]
IEBN [67]	Orthogonal [68]	
SGENet [69]		
SEGMENTATION	**DETECTION**	**BATCH NORMALIZATION**
SegNet [70]	Faster RCNN [71]	VGG
PSPNet [72]		VGG-bn
PYTORCH PRETRAIN	**MATTING**	**LEARNING RATE**
ResNet18/34/50	Deep image matting [73]	Schedule150-225
VGG11/16/19	AlphaGAN matting [74]	Schedule82-164
STYLE TRANSFER	**GAN**	Schedule60-120
Fast neural style [75]	DCGAN [76]	Cos-lr [77]

[10] The statistical tests about the situation with or without weight decay can be found in Appendix M.

Next, we show the passing rate about the statistical tests for different situations. “in the front of network” denotes whether all the failed cases are the layers whose position is in the front of the network.

For Network structure: `https://github.com/bearpaw/pytorch-classification`.

Table 7: **Network structure**.

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
ResNet164	CIFAR100	98.77%	97.55%	100%	97.55%	✓
VGG16	CIFAR100	100%	93.75%	100%	100%	✓
AlexNet	CIFAR100	100%	100%	100%	100%	✓
DenseNet-BC-100-12	CIFAR100	100%	98.99%	100%	98.99%	✓
PreResNet110	CIFAR100	100%	99.08%	100%	100%	✓
WRN28-10	CIFAR100	100%	100%	100%	100%	✓
ResNext-16x64d	CIFAR100	100%	100%	100%	100%	✓
ResNet164	CIFAR10	100.00%	97.55%	100%	97.55%	✓
VGG16	CIFAR10	100%	93.75%	100%	93.75%	✓
AlexNet	CIFAR10	100%	100%	100%	100%	✓
DenseNet-BC-100-12	CIFAR10	100%	100%	100%	98.99%	✓
PreResNet110	CIFAR10	100%	99.08%	100%	100%	✓
WRN28-10	CIFAR10	100%	100%	100%	100%	✓
ResNext-16x64d	CIFAR10	100%	100%	100%	100%	✓

For Optimizer: `https://pytorch.org/docs/master/optim.html#torch-optim`.

Table 8: **Optimizer**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
ASGD	ResNet164	100%	99.39%	99.39%	100%	✓
Adam	ResNet164	99.39%	90.18%	100%	99.39%	✗
Adagrad	ResNet164	100%	99.39%	100%	100%	✓
Adamax	ResNet164	100%	96.93%	100%	99.39%	✗
Adadelta	ResNet164	100%	100%	100%	100%	✓
SGD	ResNet164	98.77%	97.55%	100%	97.53%	✓
ASGD	VGG16	100%	100%	93.75%	100%	✓
Adam	VGG16	93.75%	93.75%	100%	100.00%	✓
Adagrad	VGG16	100%	100%	100%	100%	✓
Adamax	VGG16	100%	100%	100%	93.75%	✗
Adadelta	VGG16	100%	100%	100%	100%	✓
SGD	VGG16	100%	93.75%	100%	100%	✓
ASGD	AlexNet	100%	100%	100%	100%	✓
Adam	AlexNet	100%	100%	100%	100%	✓
Adagrad	AlexNet	100%	100%	100%	100%	✓
Adamax	AlexNet	100%	100%	100%	100%	✓
Adadelta	AlexNet	100%	100%	100%	100%	✓
SGD	AlexNet	100%	100%	100%	100%	✓

For Regularization:`https://github.com/LeungSamWai/Drop-Activation`

`https://github.com/uoguelph-mlrg/Cutout`

`https://github.com/clovaai/CutMix-PyTorch`

`https://github.com/DeepVoltaire/AutoAugment`

For Attention:`https://github.com/moskomule/senet.pytorch`

`https://github.com/gbup-group/DIANet`

`https://github.com/EvgenyKashin/SRMnet`

Table 9: **Regularization**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
L1 norm	ResNet164	100%	99.39%	99.39%	100%	✓
L2 norm	ResNet164	98.77%	97.53%	100%	97.53%	✓
RReLU	ResNet164	100%	99.39%	100%	100%	✓
Dropact	ResNet164	100%	96.93%	100%	99.39%	✓
Autoaugment	ResNet164	100%	96.93%	100%	99.39%	✓
Cutout	ResNet164	100%	100%	100%	100%	✓
Cutmix	ResNet164	98.77%	97.53%	100%	97.53%	✓
L1 norm	WRN28-10	100%	96.43%	100%	96.43%	✓
L2 norm	WRN28-10	100%	100%	100%	100%	✓
RReLU	WRN28-10	100%	96.43%	100%	100%	✓
Dropact	WRN28-10	100%	96.43%	100%	100%	✓
Autoaugment	WRN28-10	100%	96.43%	100%	100%	✓
Cutout	WRN28-10	100%	96.43%	100%	100%	✓
Cutmix	WRN28-10	100%	100%	100%	100%	✓
L1 norm	VGG16	100%	93.75%	100%	100%	✓
L2 norm	VGG16	100%	93.75%	100%	100%	✓
RReLU	VGG16	100%	93.75%	100%	93.75%	✓
Dropact	VGG16	100%	93.75%	100%	100%	✓
Autoaugment	VGG16	100%	93.75%	100%	100%	✓
Cutout	VGG16	100%	93.75%	93.75%	93.75%	✓
Cutmix	VGG16	100%	93.75%	100%	100%	✓
L1 norm	PreResNet110	100%	99.08%	100%	100%	✓
L2 norm	PreResNet110	100%	99.08%	100%	100%	✓
RReLU	PreResNet110	100%	100%	100%	100%	✓
Dropact	PreResNet110	100%	99.08%	100%	100%	✓
Autoaugment	PreResNet110	100%	100%	100%	100%	✓
Cutout	PreResNet110	100%	99.08%	99.08%	99.08%	✓
Cutmix	PreResNet110	100%	99.08%	100%	100%	✓
L1 norm	AlexNet	100%	100%	100%	100%	✓
L2 norm	AlexNet	100%	100%	100%	100%	✓
RReLU	AlexNet	100%	100%	100%	100%	✓
Dropact	AlexNet	100%	100%	100%	100%	✓
Autoaugment	AlexNet	100%	100%	100%	100%	✓
Cutout	AlexNet	100%	100%	100%	100%	✓
Cutmix	AlexNet	100%	100%	100%	100%	✓
L1 norm	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
L2 norm	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
RReLU	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
Dropact	DenseNet-BC-100-12	98.99%	98.99%	98.99%	98.99%	✓
Autoaugment	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
Cutout	DenseNet-BC-100-12	100%	98.99%	98.99%	98.99%	✓
Cutmix	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓

https://github.com/luuuyi/CBAM.PyTorch

https://github.com/gbup-group/IEBN

https://github.com/implus/PytorchInsight

Table 10: **Attention**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
SENet	ResNet164	99.39%	99.39%	100%	100%	✓
DIANet	ResNet164	99.39%	99.39%	100%	100%	✓
SRMNet	ResNet164	99.39%	97.55%	100%	99.39%	✓
CBAM	ResNet164	99.39%	99.39%	100%	100%	✓
IEBN	ResNet164	99.39%	99.39%	99.39%	99.39%	✓
SGENet	ResNet164	99.39%	98.77%	100%	100%	✓
SENet	VGG16	100%	93.75%	100%	100%	✓
DIANet	VGG16	100%	93.75%	100%	93.75%	✓
SRMNet	VGG16	100%	100%	100%	100%	✓
CBAM	VGG16	100%	93.75%	100%	100%	✓
IEBN	VGG16	100%	93.75%	93.75%	93.75%	✓
SGENet	VGG16	100%	93.75%	100%	100%	✓
SENet	PreResNet110	99.08%	100%	100%	100%	✓
DIANet	PreResNet110	100%	99.08%	100%	100%	✓
SRMNet	PreResNet110	100%	99.08%	99.08%	100%	✓
CBAM	PreResNet110	100%	100%	100%	100%	-
IEBN	PreResNet110	100%	99.08%	100%	99.08%	✓
SGENet	PreResNet110	100%	100%	100%	99.08%	✓
SENet	DenseNet-BC-100-12	100%	100%	100%	100%	✓
DIANet	DenseNet-BC-100-12	98.99%	98.99%	100%	100%	✓
SRMNet	DenseNet-BC-100-12	100%	98.99%	98.99%	98.99%	✓
CBAM	DenseNet-BC-100-12	100%	100%	100%	98.99%	✓
IEBN	DenseNet-BC-100-12	100%	98.99%	100%	100%	✓
SGENet	DenseNet-BC-100-12	100%	100%	98.99%	100%	✓
SENet	WRN28-10	100%	96.43%	100%	100%	✓
DIANet	WRN28-10	100%	96.43%	100%	100%	✓
SRMNet	WRN28-10	100%	96.43%	100%	100%	✓
CBAM	WRN28-10	100%	96.43%	100%	100%	✓
IEBN	WRN28-10	100%	96.43%	100%	100%	✓
SGENet	WRN28-10	100%	96.43%	100%	100%	✓

For initialization:

https://pytorch.org/docs/master/nn.init.html#nn-init-doc.

For dataset:

For other tasks:

https://github.com/meetshah1995/pytorch-semse

https://github.com/jwyang/faster-rcnn.pytorch

https://github.com/speedinghzl/pytorch-segmentation-toolbox

https://github.com/foamliu/Deep-Image-Matting-PyTorch

https://github.com/CDOTAD/AlphaGAN-Matting

https://github.com/abhiskk/fast-neural-style

Table 11: **Initialization**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
Kaiming-uniform	ResNet164	98.77%	97.55%	100%	100%	✓
Kaiming-normal	ResNet164	98.77%	97.53%	100%	97.55%	✓
Xavier-normal	ResNet164	98.77%	96.32%	100%	97.55%	✓
Xarier-uniform	ResNet164	98.16%	96.32%	100%	99.39%	✓
Orthogonal	ResNet164	97.55%	96.32%	100%	100%	✓
Kaiming-uniform	VGG16	100%	93.75%	100%	100%	✓
Kaiming-normal	VGG16	100%	93.75%	100%	100%	✓
Xavier-normal	VGG16	100%	93.75%	100%	93.75%	✓
Xarier-uniform	VGG16	100%	93.75%	100%	93.75%	✓
Orthogonal	VGG16	100%	93.75%	93.75%	93.75%	✓
Kaiming-uniform	WRN28-10	100%	96.43%	100%	100%	✓
Kaiming-normal	WRN28-10	100%	100%	100%	100%	✓
Xavier-normal	WRN28-10	100%	96.43%	100%	100%	✓
Xarier-uniform	WRN28-10	100%	96.43%	100%	100%	✓
Orthogonal	WRN28-10	100%	96.43%	100%	100%	✓
Kaiming-uniform	PreResNet110	100%	99.08%	100%	100%	✓
Kaiming-normal	PreResNet110	100%	99.08%	100%	100%	✓
Xavier-normal	PreResNet110	100%	100%	100%	100%	✓
Xarier-uniform	PreResNet110	100%	99.08%	100%	100%	✓
Orthogonal	PreResNet110	100%	100%	100%	100%	✓
Kaiming-uniform	AlexNet	100%	100%	100%	100%	✓
Kaiming-normal	AlexNet	100%	100%	100%	100%	✓
Xavier-normal	AlexNet	100%	100%	100%	100%	✓
Xarier-uniform	AlexNet	100%	100%	100%	100%	✓
Orthogonal	AlexNet	100%	100%	100%	100%	✓
Kaiming-uniform	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
Kaiming-normal	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
Xavier-normal	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓
Xarier-uniform	DenseNet-BC-100-12	98.99%	98.99%	98.99%	98.99%	✓
Orthogonal	DenseNet-BC-100-12	100%	98.99%	100%	98.99%	✓

Table 12: **Dataset**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
CIFAR10	WRN28-10	100%	96.43%	100%	100%	✓
CIFAR100	WRN28-10	100%	100%	100%	100%	✓
ImageNet	WRN28-10	100%	96.43%	100%	100%	✓
MINIST	WRN28-10	100%	96.43%	100%	96%	✓

https://github.com/csinva/gan-pretrained-pytorch

Table 13: **Other tasks**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
SgeNet(Cityscapes)	Segmentation	100%	100%	100%	100%	✓
PSPNet(Cityscapes)	Segmentation	100%	99.12%	100%	99.12%	✓
ResNet101(COCO)	Faster RCNN	100%	99.05%	100%	100%	✗
ResNet101(VOC2007)	Faster RCNN	100%	99.05%	100%	100%	✗
VGG16(Visual Genome)	Faster RCNN	100%	93.75%	100%	100%	✓
AlphaGAN	Image matting	100%	95.00%	100%	95.00%	✓
Deep image matting	Image matting	100%	100%	100%	100%	✓
Fast neural style	candy	86.67%	100%	100%	100%	✗
Fast neural style	mosaic	93.33%	100%	100%	100%	✓
Fast neural style	starry night	86.67%	100%	100%	100%	✗
Fast neural style	udnie	66.67%	100%	100%	100%	✗
DCGAN(MNIST)	GAN	100%	100%	100%	100%	✓
DCGAN(CIFAR10)	GAN	100%	100%	100%	100%	✓
DCGAN(CIFAR100)	GAN	100%	100%	100%	100%	✓
VGG19(CIFAR10)	without BN	100%	100%	100%	100%	✓
VGG19(CIFAR10)	with BN	93.75%	100%	100%	100%	✓
VGG19(CIFAR10-lr)	schedule(82-164)	93.75%	100%	100%	100%	✓
VGG19(CIFAR10-lr)	schedule(60-120)	93.75%	100%	100%	100%	✓
VGG19(CIFAR10-lr)	coslr	93.75%	100%	100%	100%	✓

For pytorch pretrain:http://pytorch.org/docs/master/torchvision/index.html.

Table 14: **Pytorch pretrian**

Experiments	Remark	Gaussian	Variance	Mean	Magnitude	in the front of network?
VGG11	ImageNet	100%	75.00%	100%	75.00%	✓
VGG16	ImageNet	100%	84.62%	100%	100%	✓
VGG19	ImageNet	100%	87.50%	100%	100%	✓
ResNet18	ImageNet	100%	88.24%	100%	100%	✓
ResNet34	ImageNet	100%	88.24%	100%	96.97%	✓
ResNet50	ImageNet	100%	83.67%	100%	100%	✗

Q Training through slimming

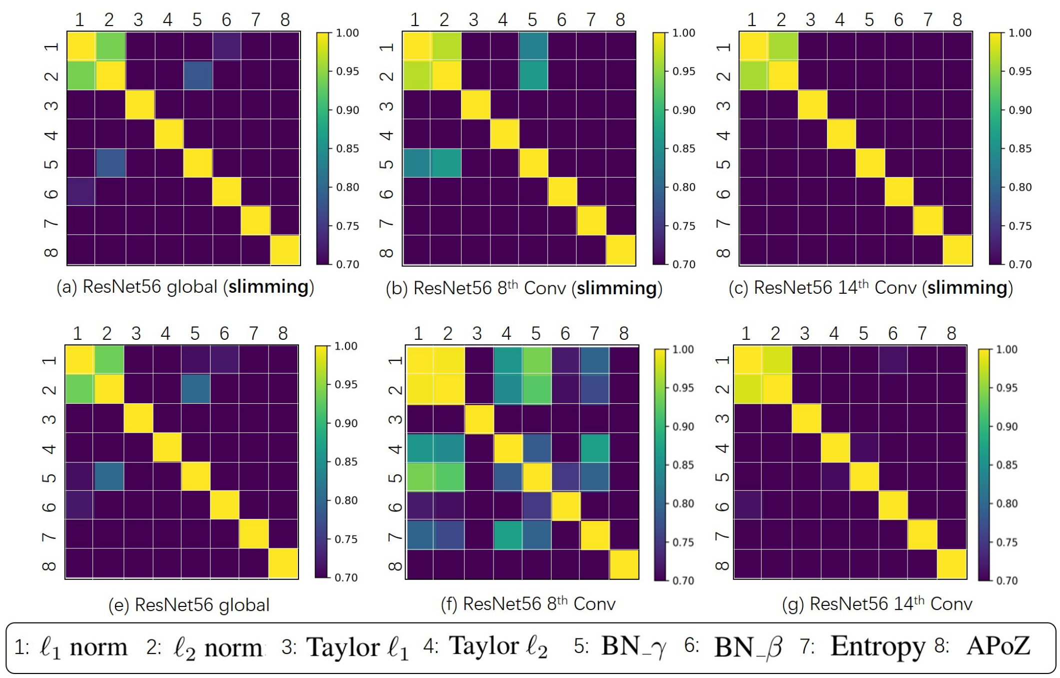


Figure 19: The Similarity for different criteria with/without slimming [34].

As a representative of the BN-based pruning method, slimming pruning[34] can not be directly compared with the criteria mentioned in the paper because it adopts a special training method. Therefore, we use the training method in [34] to train another ResNet56 on cifar100. Then, the analysis of similarities between 8 different pruning criteria on such a model is shown in Fig. 19.

In this situation, the fifth criterion BN_γ is the method introduced in [34]. From Fig. 19, there is no significant difference in the result of the similarity between ResNet56 obtained by slimming method and resnet56 trained in general.

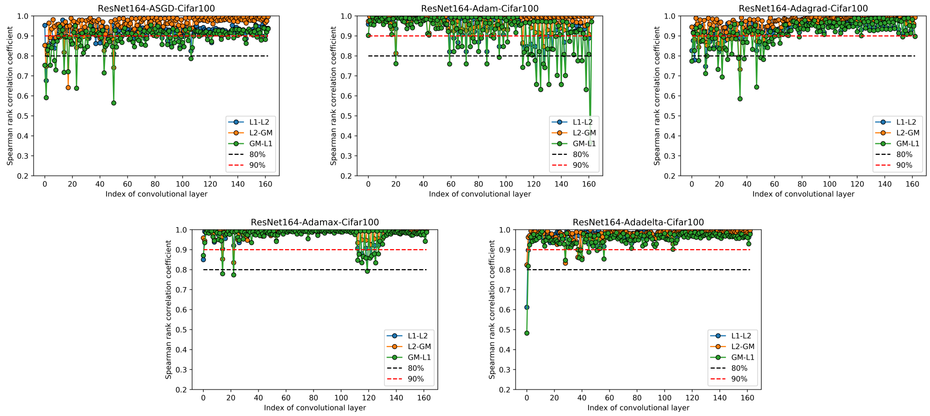


Figure 21: Optimizer

R More experiments of Sp in Norm-based criteria

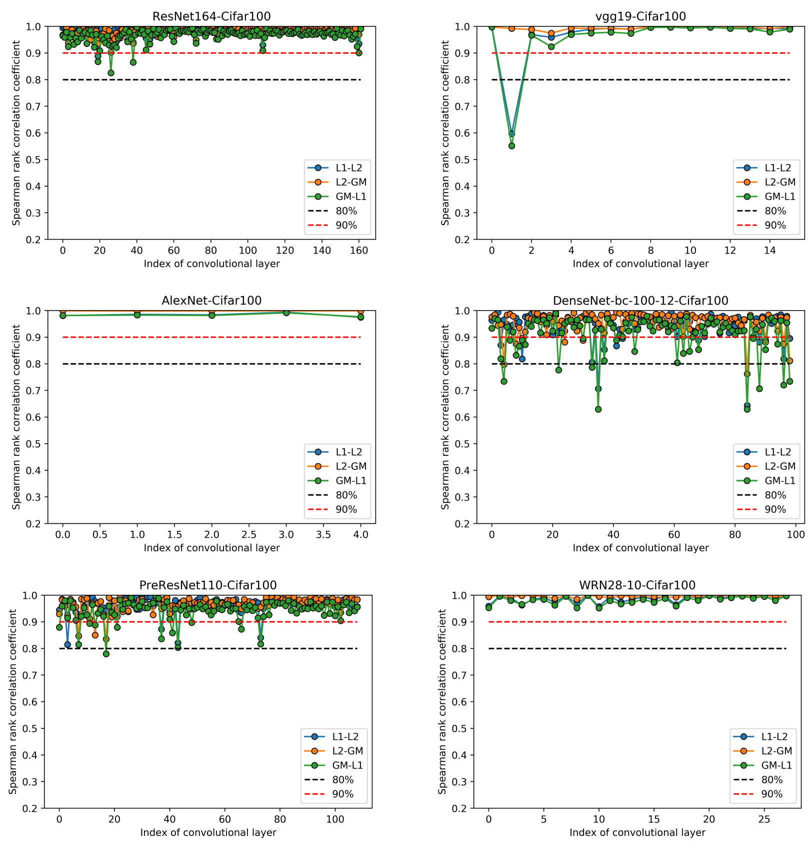


Figure 20: Network Structure

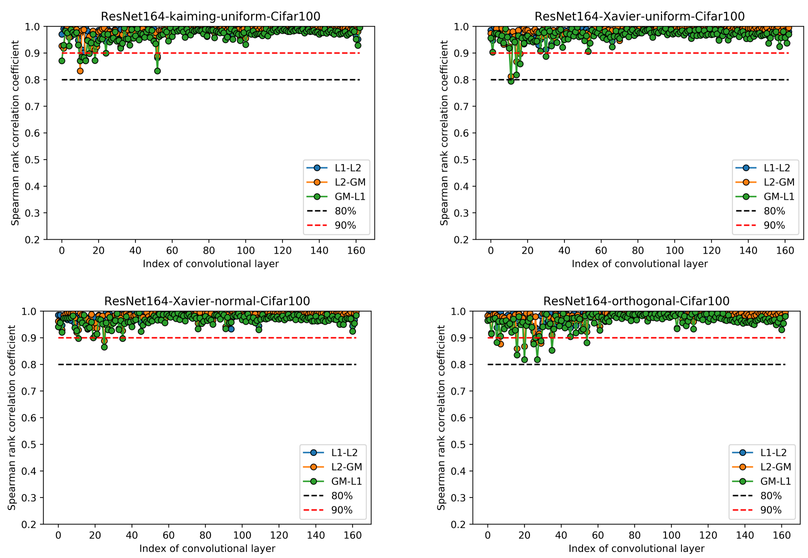


Figure 22: Initialization

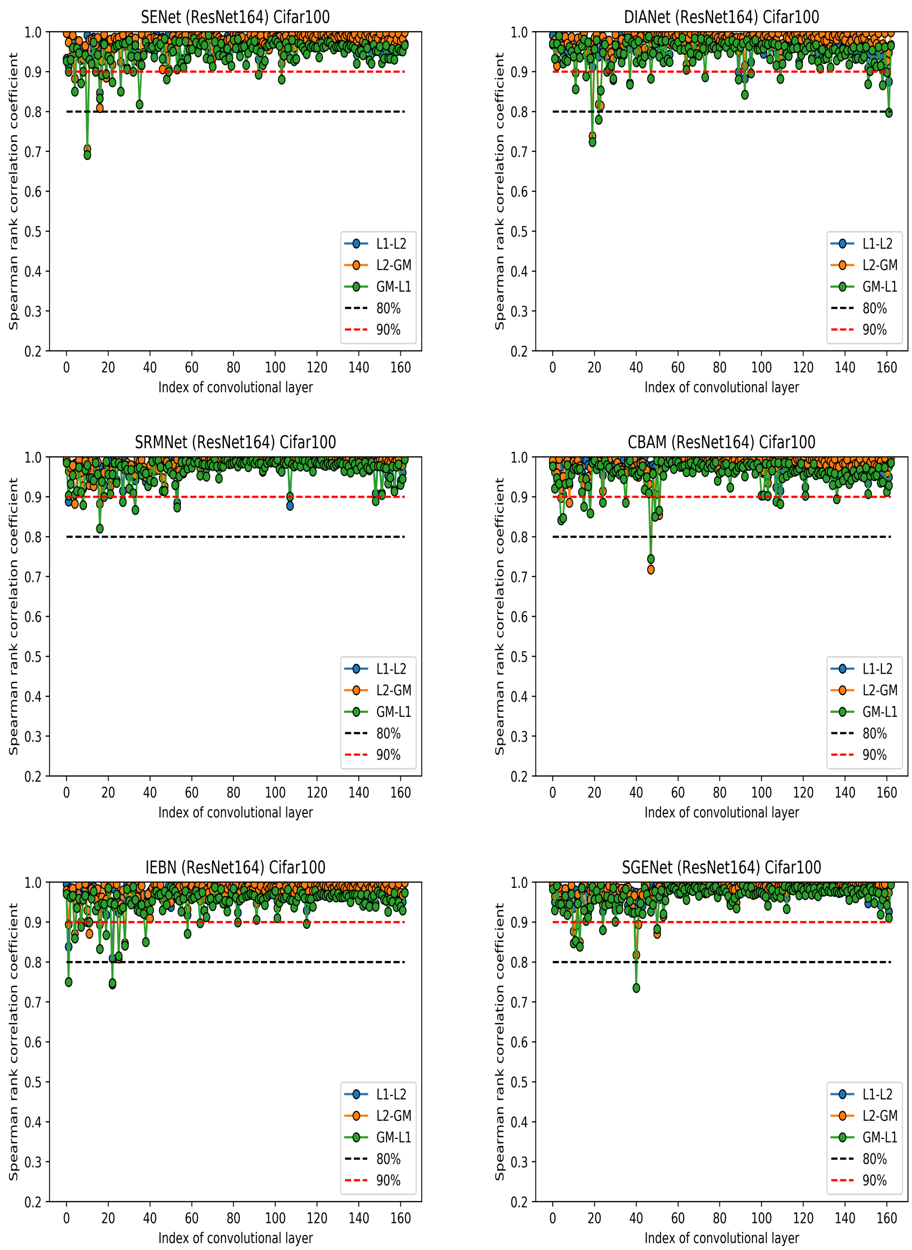


Figure 23: Attention mechanism

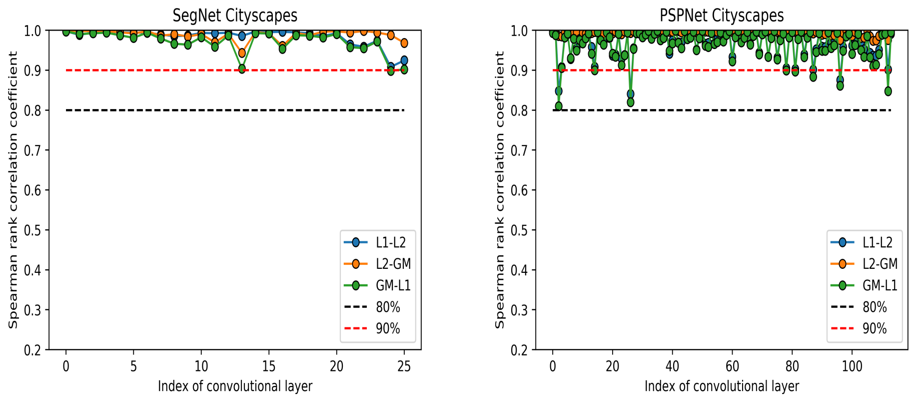


Figure 24: Other task: segmentation

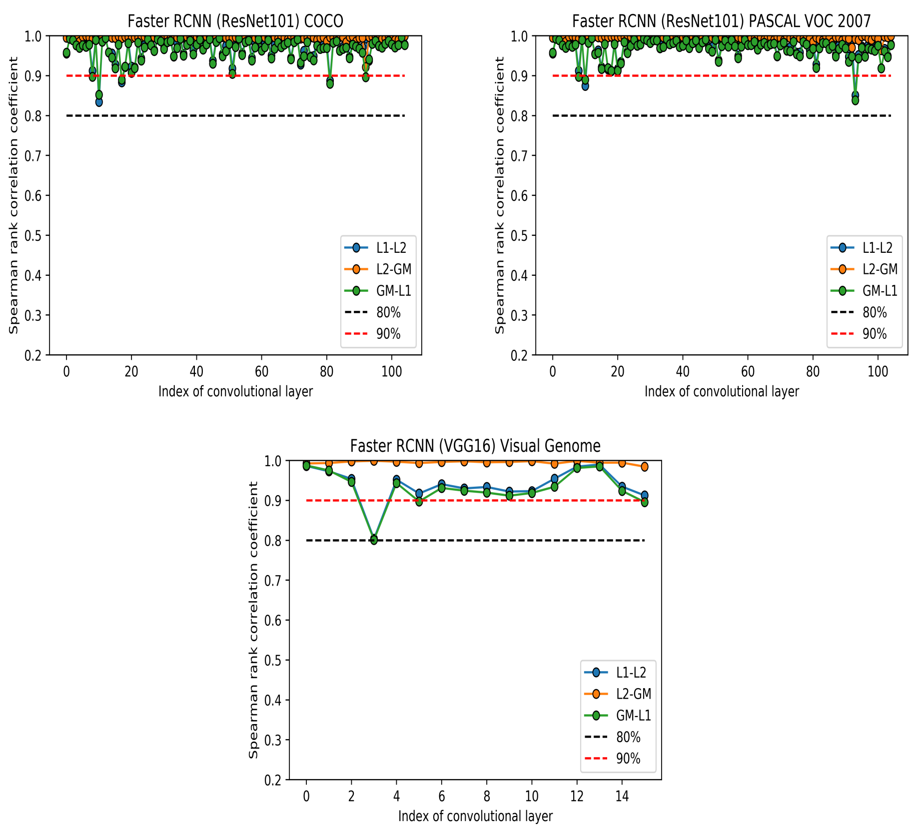


Figure 25: Other task: Faster RCNN

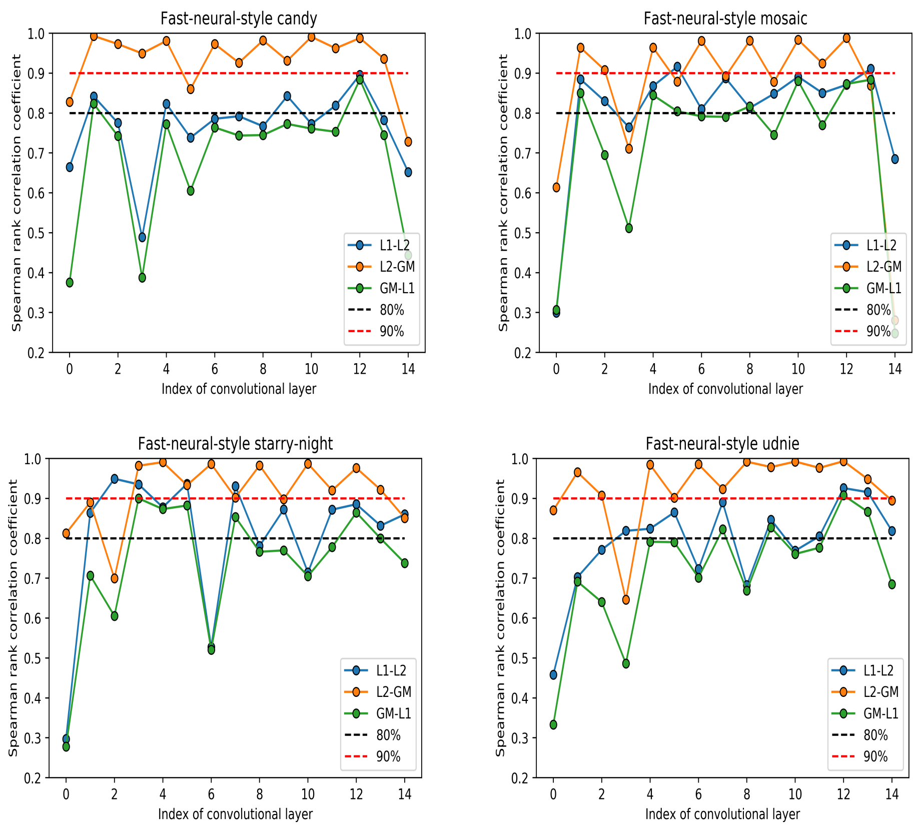


Figure 26: Other task: style transfer

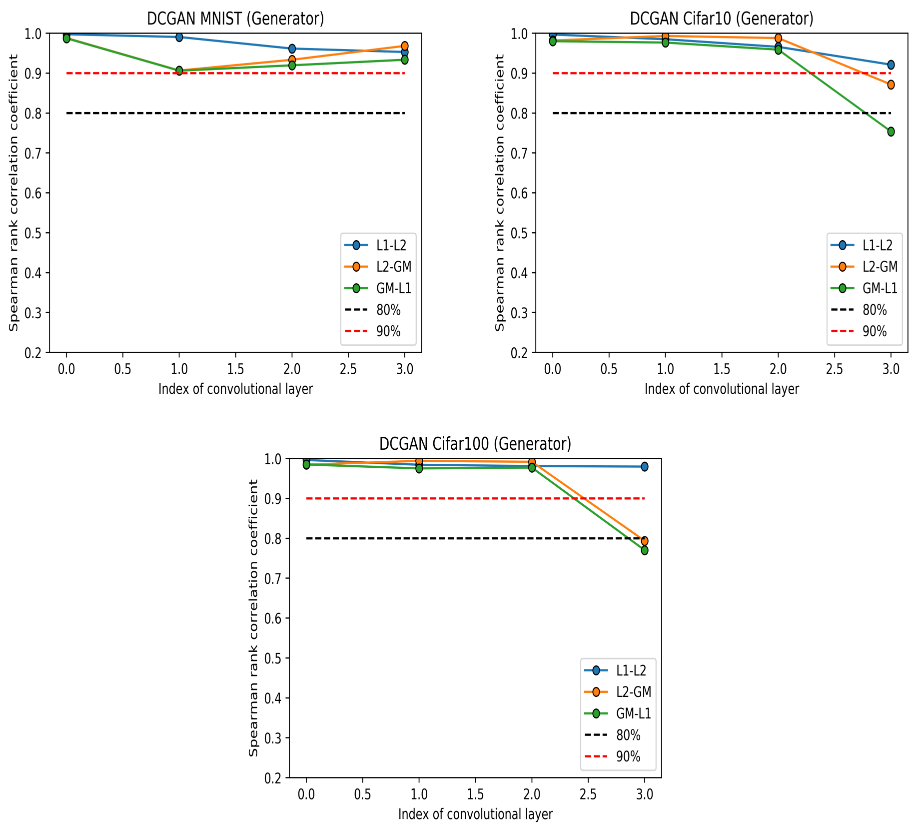


Figure 27: Other task: GAN

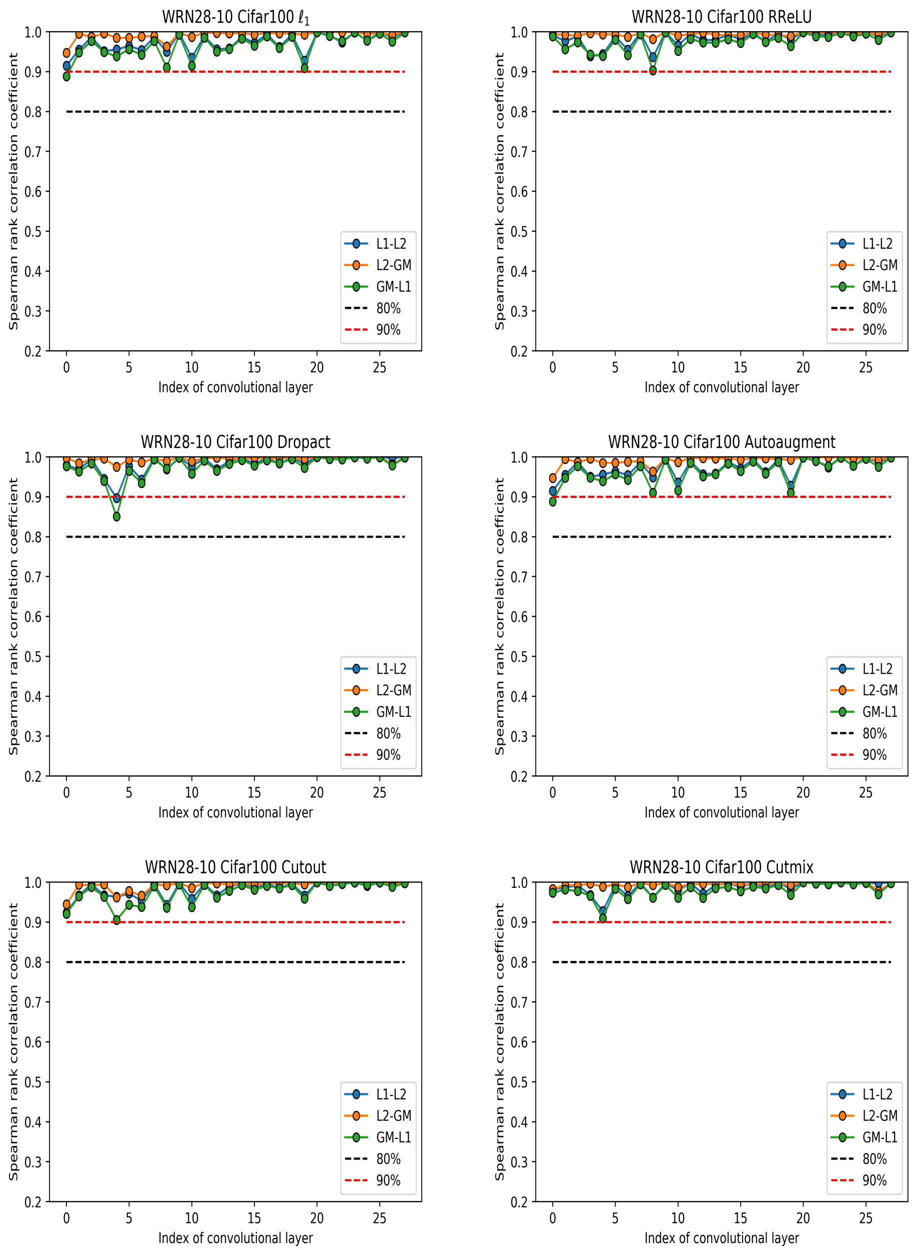


Figure 28: Other task: Regularization

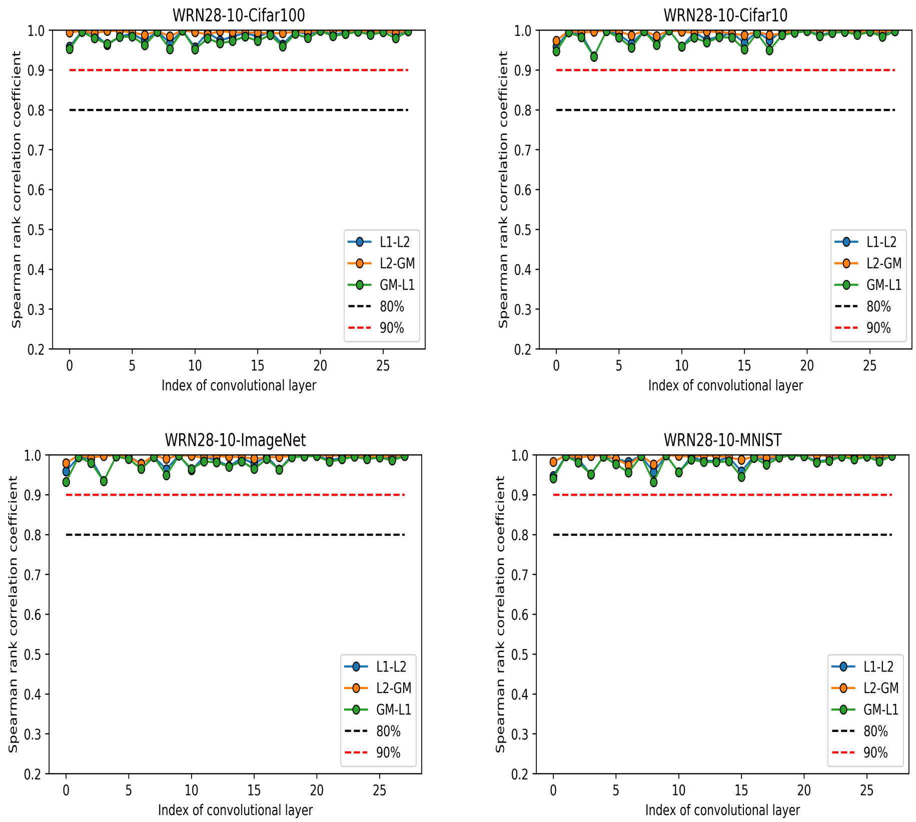


Figure 29: Dataset

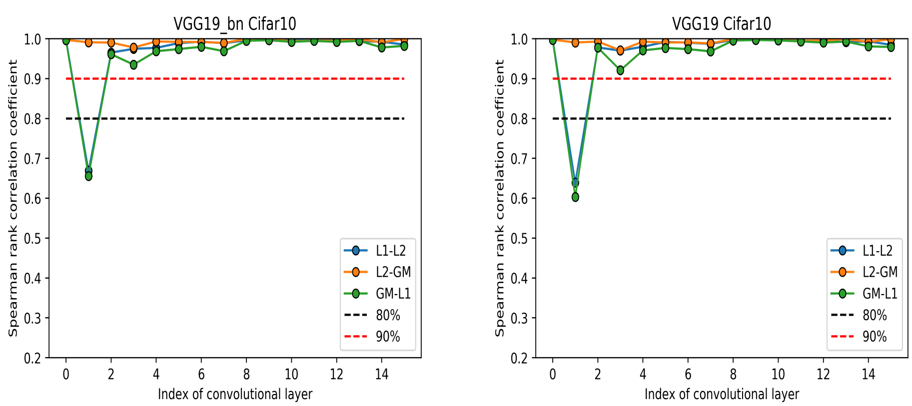


Figure 30: Batch normalization

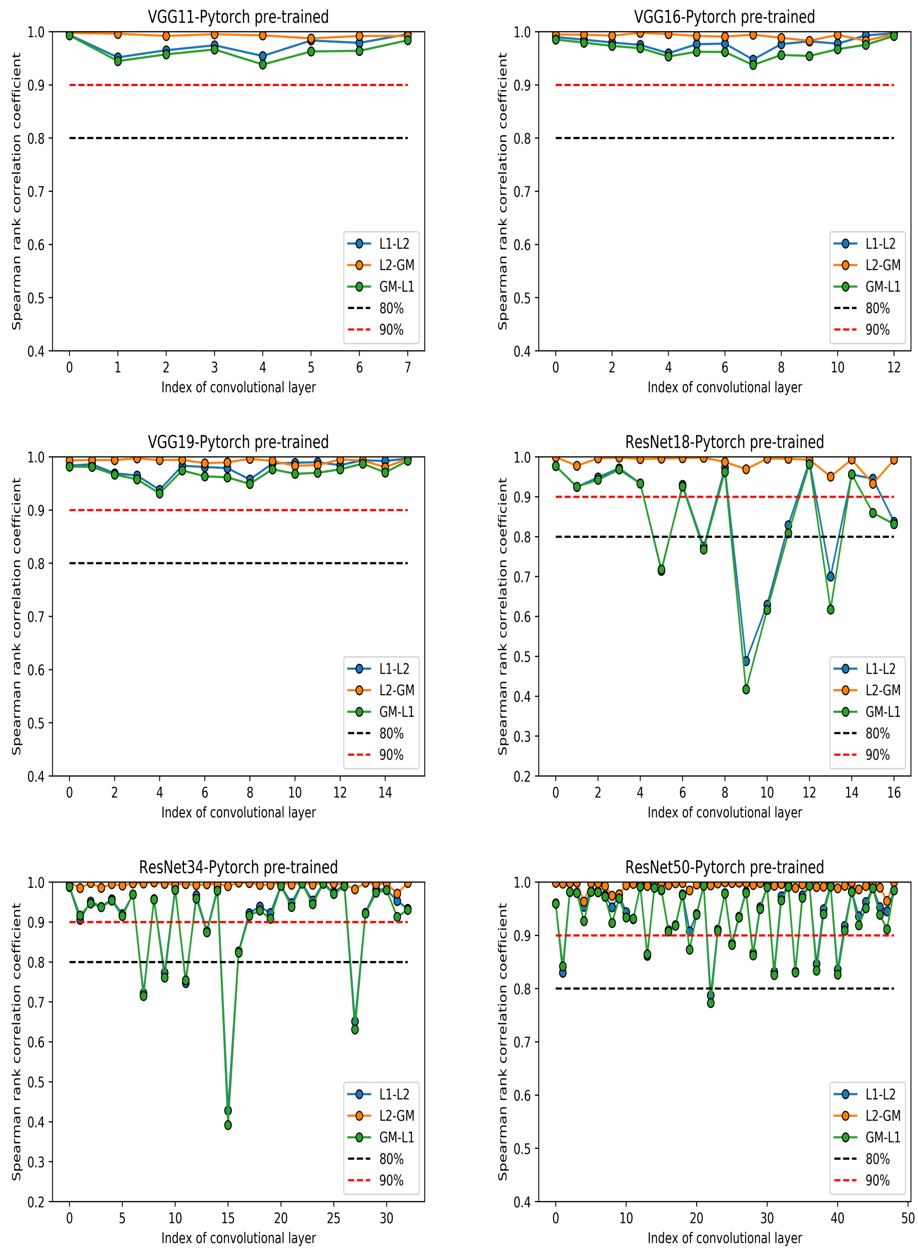


Figure 31: Pytorch pre-trained Model

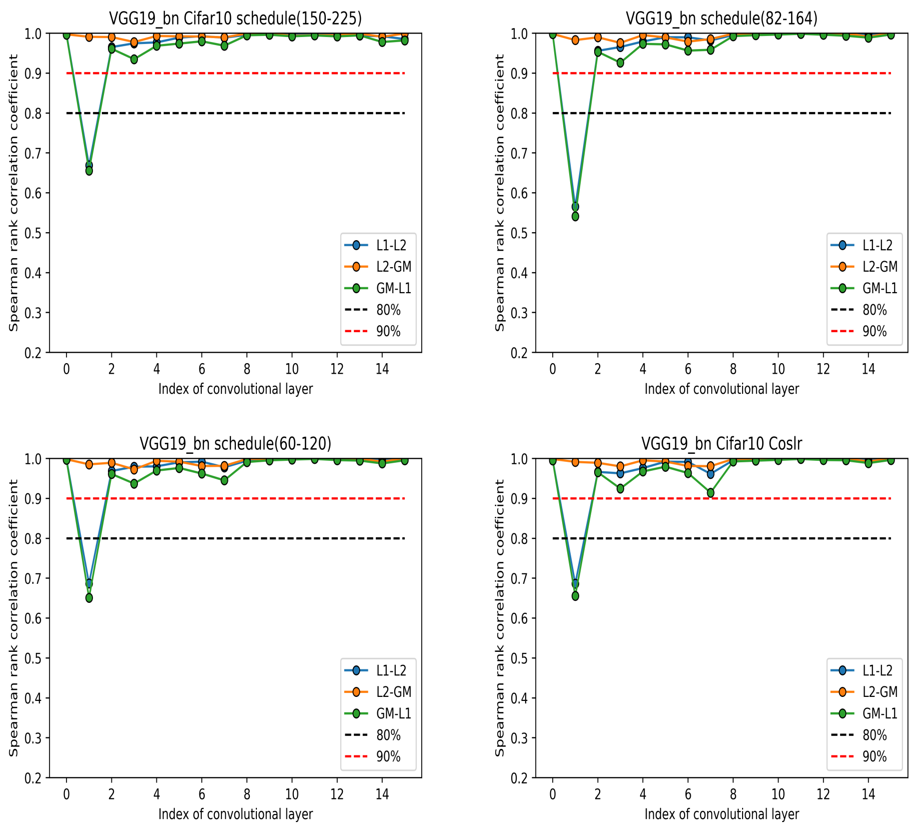


Figure 32: Learning rate