

1 **R2 & R5: "Threshold policies are optimal"**. We thank **R2, R5** for the reference (Corbett-Davies et al., 2017). Our Thm
2 1 is indeed similar. We will be sure to cite this work and explain the relationship with ours.

3 **R2 & R5: "COMPAS experiments"**. We share the same
4 reservation in using this dataset and lending validity to Con-
5 dition 1A, even though our purpose was only to show the
6 flexibility of our framework. As an alternative, we've run
7 experiments on COMPAS under all conditions (1A-D). Table
8 1 shows Prop 1 holds under 1A-B, no oscillation under 1B-
9 C, and more uncertainty under 1C-D, which is discussed in
10 Appendix F. We will replace the original version with these
11 results, or, if this still does not address reviewers' concerns,
12 we are fine with removing this experiment altogether.

13 **R2 & R3: "1D feature space"**. We thank **R2** and **R3** for suggestions on notations. To clarify, our work is not limited to
14 1D feature space (line 115-116); the generalization to \mathbb{R}^d is in Appendix E.

15 **R2, R3 & R5: "Interventions"**. **@R2 sensitivity**: This is one of our main findings (lines 7-9) and reflects broader
16 challenges in designing a fair policy: the effect of intervention highly depends on problem parameters, and the same
17 intervention may lead to contrarian results as parameters change. **@R3 Trade-off**: Prop. 1 improves $\hat{\alpha}^s$ by sacrificing
18 instant utility; Prop. 2 achieves equality but violates static fairness and sacrifices instant utility. Note that the sacrifice
19 in instant utility in both cases may actually result in improved long-term total utility (line 283). While T_{yd}^s is a group
20 property, it is controllable through community-level interventions (Prop. 3) such as social support (subsidy, training, etc.)
21 to sub-populations (line 294). **@R5 usage**: Intervention remains feasible even without knowing the true qualification
22 (of the rejected population); e.g., supporting all those rejected (or accepted) would increase both T_{1d} and T_{0d} .

23 **R2: "Related work in economics"**. The model studied in (Coate and Loury, 1993) is more relevant to [33], where people
24 manipulate their qualifications and groups have identical feature distribution and response to the policy. We will add
25 this comparison and more related works in economics. **"Limitations on group-specific policies"**. In some cases the use
26 of sensitive attribute is allowed (e.g., per ECOA Regulation B, *age* can be used in lending in the US). Nonetheless, we
27 will clarify that group-specific policies may not be generally applicable.

28 **R3: "Stability of unique equilibrium"**. It is true there can be oscillation under conditions in Thm.3, as they only guarantee
29 uniqueness but not stability. We discuss stability in Appendix F (line 662-677), and will clarify this in the main body.
30 **"Harm/benefit of fairness if natural equality is not broken"**. This is examined in Thm.4 (line 218-219): equality is
31 violated/maintained if distributions are different/same. **"Scenarios under natural inequality"**. Under our model two groups
32 can be different in transition or/and feature distribution. Natural inequality arises when either one or both differ across
33 two groups. We thus regard these as two sources of inequality, consider them separately by fixing one and varying the
34 other (two scenarios we studied), and examine whether fairness constraints can address inequality caused by each (line
35 228-230). **"When DP flips the advantaged group"**. In this case the gap $|\hat{\alpha}_{DP}^a - \hat{\alpha}_{DP}^b|$ highly depends on feature distribution.
36 Empirical results (Table 3, Appendix A) show DP can reduce this gap (mitigate inequality): $\hat{\alpha}_{DP}^b - \hat{\alpha}_{DP}^a < \hat{\alpha}_{UN}^a - \hat{\alpha}_{UN}^b$.

37 **R3 & R5: "Markov dynamics"**. As long as an appropriate "state" (sufficient statistics) can be identified, the Markov
38 assumption holds; this in the worst case would be the entire history which would indeed be undesirable. In practice
39 historical information is often summarized into a (pseudo) sufficient statistics to enable tractable decision making; e.g.,
40 lending decisions rely on the entire history only through summaries such as the credit score or a set of scores, which
41 can be regarded as the state in a Markov process. **@R3 linearity**: Although individual action doesn't depend on α_t^s , we
42 note Eqn. (4) is *not* linear in qualification α_t^s , as g_t^{1s} and g_t^{0s} are functions of policy π_t^s , which is nonlinear in α_t^s .

43 **R3 & R5: "Justification on modeling choices"**. **@R3**: In addition to current explanations and examples (line 162-165 on
44 transitions; line 234-236, 254-255 on two scenarios under natural inequality), we will further strengthen the motivation
45 for these modeling choices. **@R5 POMDP**: We think POMDP is a reasonable framework to capture the sequential
46 nature of the problem and the fact the true qualifications Y are unobservable to the decision maker, and the decision is
47 based on X and a belief state on Y . A (PO)MDP approach has been motivated and used in similar studies [9,23]. In
48 particular, [9] shows that although many works on fairness didn't explicitly use (PO)MDP to model dynamics, they
49 can all be cast into the standard framework of (PO)MDP, such as works on lending [32,36], college admission [22,28],
50 attention allocation [11], etc. The studies on the scenario mentioned by **R5** (outcomes are observed only under positive
51 decisions) are orthogonal to our work, which we introduce and discuss in Appendix C (line 596-598).

52 **R5: "Conditions 1A-D"**. Under our POMDP framework (Fig. 1), dynamics of α_t follow Eqn. (4) and 1A-1D actually
53 capture all possibilities including the case mentioned by the reviewer. Specifically, if $\text{avg}(T_{01} + T_{11}) \leq$ (resp. \geq)
54 $\text{avg}(T_{00} + T_{10})$ holds, then either 1A (resp. 1B) or 1C or 1D must hold. **"Explain conditions in Thm. 3"**. This can be
55 considered as a weaker version of the Lipschitz condition. More discussion is in Appendix F (line 662-677).

Table 1: $\text{osi}^*/\text{osi}_H/\text{osi}_L$ is the percentage that oscillation occurs among 125 set of different transitions under policy $\text{UN}^*/\text{UN}_{\theta_H}/\text{UN}_{\theta_L}$. Among transitions that lead to stable equilibrium, Col 2/Col 3 shows the percentage that $\text{UN}_{\theta_H}/\text{UN}_{\theta_L}$ results in lower recidivism compared with UN^* .

	$\hat{\alpha}_{\theta_H} < \hat{\alpha}^*$	$\hat{\alpha}_{\theta_L} < \hat{\alpha}^*$	osi^*	osi_H	osi_L
<i>A</i>	0	1	0.29	0.12	0.36
<i>B</i>	0.99	0.01	0	0	0
<i>C</i>	0.37	0.28	0	0	0
<i>D</i>	0.79	0.63	0.06	0	0.13