Supplementary Materials

A Further Specification of Experiments

Following [1], we consider a 5-way 5-shot task on both the FC100 and miniImageNet datasets, where we evaluate the model's ability to discriminate 5 unseen classes, given only 5 labelled samples per class. We adopt Adam [15] as the optimizer for the meta outer-loop update, and adopt the cross-entropy loss to measure the error between the predicted and true labels.

A.1 Introduction of FC100 and miniImageNet datasets

FC100 dataset. The FC100 dataset [23] is generated from CIFAR-100 [17], and consists of 100 classes with each class containing 600 images of size 32. Following recent work [23, 18], we split these 100 classes into 60 classes for meta-training, 20 classes for meta-validation, and 20 classes for meta-testing.

miniImageNet dataset. The miniImageNet dataset [30] consists of 100 classes randomly chosen from ImageNet [27], where each class contains 600 images of size 84×84. Following the repository [1], we partition these classes into 64 classes for meta-training, 16 classes for meta-validation, and 20 classes for meta-testing.

A.2 Model Architectures and Hyper-Parameter Setting

We adopt the following four model architectures depending on the dataset and the geometry of the inner-loop loss. The hyper-parameter configuration for each architecture is also provided as follows.

Case 1: FC100 dataset, strongly-convex inner-loop loss. Following [1], we use a 4-layer CNN of four convolutional blocks, where each block sequentially consists of a 3×3 convolution with a padding of 1 and a stride of 2, batch normalization, ReLU activation, and 2×2 max pooling. Each convolutional layer has 64 filters. This model is trained with an inner-loop stepsize of 0.005, an outer-loop (meta) stepsize of 0.001, and a mini-batch size of $B = 32$. We set the regularization parameter λ of the L^2 regularizer to be $\lambda = 5$.

Case 2: FC100 dataset, nonconvex inner-loop loss. We adopt a 5-layer CNN with the first four convolutional layers the same as in **Case 1**, followed by ReLU activation, and a full-connected layer with size of $256 \times$ ways. This model is trained with an inner-loop stepsize of 0.04, an outer-loop (meta) stepsize of 0.003, and a mini-batch size of $B = 32$.

Case 3: miniImageNet dataset, strongly-convex inner-loop loss. Following [24], we use a 4-layer CNN of four convolutional blocks, where each block sequentially consists of a 3×3 convolution with 32 filters, batch normalization, ReLU activation, and 2×2 max pooling. We choose an inner-loop stepsize of 0.002, an outer-loop (meta) stepsize of 0.002, and a mini-batch size of $B = 32$, and set the regularization parameter λ of the L^2 regularizer to be $\lambda = 0.1$.

Case 4: miniImageNet dataset, nonconvex inner-loop loss. We adopt a 5-layer CNN with the first four convolutional layers the same as in **Case 3**, followed by ReLU activation, and a full-connected layer with size of $128 \times$ ways. We choose an inner-loop stepsize of 0.02, an outer-loop (meta) stepsize of 0.003, and a mini-batch size of $B = 32$.

A.3 Experiments with SGD Optimizer

The experiments in Section 4.1 and Section 4.2 adopt the Adam optimizer. In this subsection, we conduct experiments using mini-batch stochastic gradient descent (SGD) on FC100 dataset. For both the strongly-convex and nonconvex cases, we choose an inner-loop stepsize of 0.05, an outer-loop (meta) stepsize of 0.05, and a mini-batch size of $B = 32$. The results are given in Figure 3. It can be seen that the nature of the results remains the same as those done with the Adam optimizer.

A.4 Experiments on Comparison of ANIL and MAML

In Figure 4, we compare the computational efficiency between ANIL and MAML. For the miniImageNet dataset, we choose the inner-loop stepsize as 0.1, the outer-loop (meta) stepsize as 0.002, the

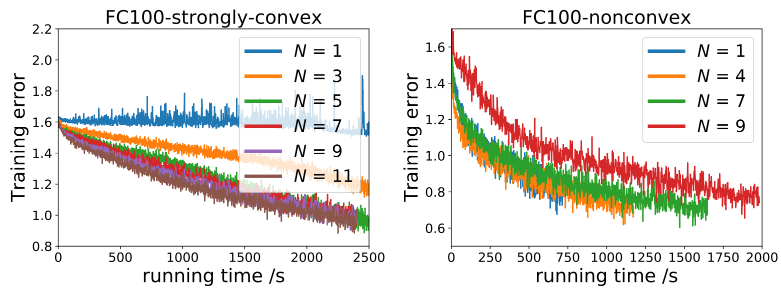
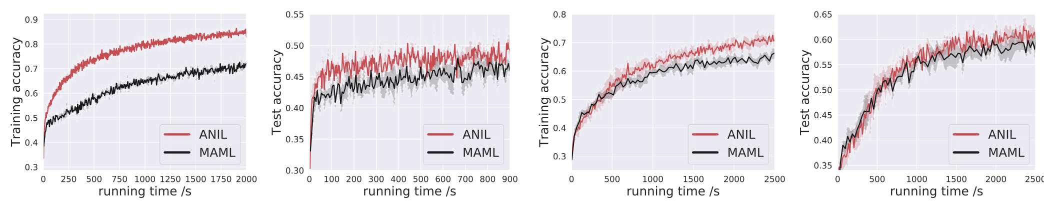


Figure 3: Convergence of ANIL with mini-batch SGD over FC100 dataset. Left plot: strongly-convex inner-loop loss; right plot: nonconvex inner-loop loss.

mini-batch size as 32, and the number of inner-loop steps as 5 for ANIL. For MAML, we choose the inner-loop stepsize as 0.5, the outer-loop stepsize as 0.003, the mini-batch size as 32, and the number of inner-loop steps as 3. For the FC100 dataset, we choose the inner-loop stepsize as 0.1, the outer-loop (meta) stepsize as 0.001, the mini-batch size as 32 for ANIL. For MAML, we choose the inner-loop stepsize as 0.5, the outer-loop stepsize as 0.001, and the mini-batch size as 32. We choose the number of inner-loop steps as 10 for ANIL and 3 for MAML. It can be seen that ANIL converges faster than MAML, as well supported by our theoretical results.



(a) dataset: FC100 (b) dataset: miniImageNet

Figure 4: Computational comparison of ANIL and MAML. For each dataset, left plot: training accuracy v.s. running time; right plot: test accuracy v.s. running time.

B Proof of Proposition 1

We first prove the form of the partial gradient $\frac{\partial L_{\mathcal{D}_i}(w^i_{k,N},\phi_k)}{\partial w_k}$. Using the chain rule, we have

$$\begin{aligned}\frac{\partial L_{\mathcal{D}_i}(w^i_{k,N},\phi_k)}{\partial w_k} &= \frac{\partial w^i_{k,N}(w_k,\phi_k)}{\partial w_k}\nabla_w L_{\mathcal{D}_i}(w^i_{k,N},\phi_k) + \frac{\partial \phi_k}{\partial w_k}\nabla_\phi L_{\mathcal{D}_i}(w^i_{k,N},\phi_k)\\ &= \frac{\partial w^i_{k,N}(w_k,\phi_k)}{\partial w_k}\nabla_w L_{\mathcal{D}_i}(w^i_{k,N},\phi_k),\end{aligned} \tag{3}$$

where the last equality follows from the fact that $\frac{\partial \phi_k}{\partial w_k} = 0$. Recall that the gradient updates in Algorithm 1 are given by

$$w^i_{k,m+1} = w^i_{k,m} - \alpha\nabla_w L_{\mathcal{S}_i}(w^i_{k,m},\phi_k),\; m = 0,1,...,N-1, \tag{4}$$

where $w^i_{k,0} = w_k$ for all i. Taking derivatives w.r.t. w_k in eq. (4) yields

$$\frac{\partial w^i_{k,m+1}}{\partial w_k} = \frac{\partial w^i_{k,m}}{\partial w_k} - \alpha\frac{\partial w^i_{k,m}}{\partial w_k}\nabla^2_w L_{\mathcal{S}_i}(w^i_{k,m},\phi_k) - \underbrace{\alpha\frac{\partial \phi_k}{\partial w_k}\nabla_\phi\nabla_w L_{\mathcal{S}_i}(w^i_{k,m},\phi_k)}_{0}. \tag{5}$$

Telescoping eq. (5) over m from 0 to $N-1$ yields

$$\frac{\partial w^i_{k,N}}{\partial w_k} = \prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_{k,m},\phi_k)),$$

which, in conjunction eq. (3), yields the first part in Proposition 1.

For the second part, using chain rule, we have

$$\frac{\partial L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)}{\partial \phi_k}=\frac{\partial w_{k,N}^i}{\partial \phi_k}\nabla_w L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)+\nabla_\phi L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k). \tag{6}$$

Taking derivates w.r.t. ϕ_k in eq. (4) yields

$$\begin{aligned}\frac{\partial w_{k,m+1}^i}{\partial \phi_k}=&\frac{\partial w_{k,m}^i}{\partial \phi_k}-\alpha\Big(\frac{\partial w_{k,m}^i}{\partial \phi_k}\nabla_w^2 L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k)+\nabla_\phi\nabla_w L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k)\Big)\\ =&\frac{\partial w_{k,m}^i}{\partial \phi_k}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k))-\alpha\nabla_\phi\nabla_w L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k).\end{aligned}$$

Telescoping the above equality over m from 0 to $N-1$ yields

$$\begin{aligned}\frac{\partial w_{k,N}^i}{\partial \phi_k}=&\frac{\partial w_{k,0}^i}{\partial \phi_k}\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k))\\ &-\alpha\sum_{m=0}^{N-1}\nabla_\phi\nabla_w L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k)\prod_{j=m+1}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{k,j}^i,\phi_k)),\end{aligned}$$

which, in conjunction with the fact that $\frac{\partial w_{k,0}^i}{\partial \phi_k}=\frac{\partial w_k}{\partial \phi_k}=0$ and eq. (6), yields the second part.

C Proof in Section 3.1: Strongly-Convex Inner Loop

C.1 Auxiliary Lemma

The following lemma characterizes a bound on the difference between $w_t^i(w_1,\phi_1)$ and $w_t^i(w_2,\phi_2)$, where $w_t^i(w,\phi)$ corresponds to the t^{th} inner-loop iteration starting from the initialization point (w,ϕ).

Lemma 1. *Choose α such that $1-2\alpha\mu+\alpha^2L^2>0$. Then, for any two points $(w_1,\phi_1),(w_2,\phi_2)\in\mathbb{R}^n$, we have*

$$\left\|w_t^i(w_1,\phi_1)-w_t^i(w_2,\phi_2)\right\|\leq(1-2\alpha\mu+\alpha^2L^2)^{\frac{t}{2}}\|w_1-w_2\|+\frac{\alpha L\|\phi_1-\phi_2\|}{1-\sqrt{1-2\alpha\mu+\alpha^2L^2}}.$$

Proof. Based on the updates in eq. (2), we have

$$\begin{aligned}w_{m+1}^i(w_1,\phi_1)-w_{m+1}^i(w_2,\phi_2)=&\,w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2)\\ &-\alpha\big(\nabla_w L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_1)\big)\\ &+\alpha\big(\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2)-\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_1)\big),\end{aligned}$$

which, together with the triangle inequality and Assumption 1, yields

$$\begin{aligned}&\|w_{m+1}^i(w_1,\phi_1)-w_{m+1}^i(w_2,\phi_2)\|\\ &\leq\underbrace{\left\|w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2)-\alpha\big(\nabla_w L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_1)\big)\right\|}_{P}\\ &\quad+\alpha L\|\phi_1-\phi_2\|.\end{aligned}\tag{7}$$

Our next step is to upper-bound the term P in eq. (7). Note that

$$\begin{aligned}P^2=&\|w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2)\|^2+\alpha^2\|\nabla_w L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_1)\|^2\\ &-2\alpha\Big\langle w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2),\nabla_w L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_1)\Big\rangle\\ \leq&(1+\alpha^2L^2-2\alpha\mu)\|w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2)\|^2,\end{aligned}\tag{8}$$

where the last inequality follows from the strong-convexity of the loss function $L_{\mathcal{S}_i}(\cdot,\phi)$ that for any w,w' and ϕ,

$$\langle w-w',\nabla_w L_{\mathcal{S}_i}(w,\phi)-\nabla_w L_{\mathcal{S}_i}(w',\phi)\rangle\geq\mu\|w-w'\|^2.$$

Substituting eq. (8) into eq. (7) yields

$$\|w^i_{m+1}(w_1,\phi_1) - w^i_{m+1}(w_2,\phi_2)\| \leq \sqrt{1+\alpha^2L^2-2\alpha\mu}\|w^i_m(w_1,\phi_1) - w^i_m(w_2,\phi_2)\| \\ + \alpha L\|\phi_1-\phi_2\|. \tag{9}$$

Telescoping the above inequality over m from 0 to $t-1$ completes the proof. □

C.2 Proof of Proposition 2

Using an approach similar to the proof of Proposition 1, we have

$$\frac{\partial L_{\mathcal{D}_i}(w^i_N,\phi)}{\partial w} = \prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m,\phi))\nabla_w L_{\mathcal{D}_i}(w^i_N,\phi). \tag{10}$$

Let $w^i_m(w,\phi)$ denote the m^{th} inner-loop iteration starting from (w,ϕ). Then, we have

$$\begin{aligned}
&\Big\|\frac{\partial L_{\mathcal{D}_i}(w^i_N,\phi)}{\partial w}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w^i_N,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\Big\| \\
\leq & \underbrace{\Big\|\prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_2,\phi_2),\phi_2))\Big\|\Big\|\nabla_w L_{\mathcal{D}_i}(w^i_N(w_1,\phi_1),\phi_1) - \nabla_w L_{\mathcal{D}_i}(w^i_N(w_2,\phi_2),\phi_2)\Big\|}_{P} \\
&+ \underbrace{\Big\|\prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_1,\phi_1),\phi_1))\nabla_w L_{\mathcal{D}_i}(w^i_N(w_1,\phi_1),\phi_1) - \prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_2,\phi_2),\phi_2))\nabla_w L_{\mathcal{D}_i}(w^i_N(w_1,\phi_1),\phi_1)\Big\|}_{Q},
\end{aligned} \tag{11}$$

where $w^i_m(w,\phi)$ is obtained through the following gradient descent steps

$$w^i_{t+1}(w,\phi) = w^i_t(w,\phi) - \alpha\nabla_w L_{\mathcal{S}_i}(w^i_t(w,\phi),\phi),\ t=0,...,m-1 \text{ and } w^i_0(w,\phi)=w. \tag{12}$$

We next upper-bound the term P in eq. (11). Based on the strongly-convexity of the function $L_{\mathcal{S}_i}(\cdot,\phi)$, we have $\|I - \alpha\nabla^2_w L_{\mathcal{S}_i}(\cdot,\phi)\| \leq 1-\alpha\mu$, and hence

$$\begin{aligned}
P \leq& (1-\alpha\mu)^N\|\nabla_w L_{\mathcal{D}_i}(w^i_N(w_1,\phi_1),\phi_1) - \nabla_w L_{\mathcal{D}_i}(w^i_N(w_2,\phi_2),\phi_2)\| \\
\overset{(i)}{\leq}& (1-\alpha\mu)^N L(\|w^i_N(w_1,\phi_1) - w^i_N(w_2,\phi_2)\| + \|\phi_1-\phi_2\|) \\
\overset{(ii)}{\leq}& (1-\alpha\mu)^N L\Big((1-2\alpha\mu+\alpha^2L^2)^{\frac{N}{2}}\|w_1-w_2\| + \frac{\alpha L\|\phi_1-\phi_2\|}{1-\sqrt{1-2\alpha\mu+\alpha^2L^2}} + \|\phi_1-\phi_2\|\Big) \\
\overset{(iii)}{\leq}& (1-\alpha\mu)^{\frac{3N}{2}}L\|w_1-w_2\| + (1-\alpha\mu)^N L\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|,
\end{aligned} \tag{13}$$

where (i) follows from Assumption 1, (ii) follows from Lemma 1, and (iii) follows from the fact that $\alpha\mu = \frac{\mu^2}{L^2} = \alpha^2L^2$ and $\sqrt{1-x} \leq 1-\frac{1}{2}x$.

To upper-bound the term Q in eq. (11), we have

$$Q \leq M\underbrace{\Big\|\prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_1,\phi_1),\phi_1)) - \prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_2,\phi_2),\phi_2))\Big\|}_{P_{N-1}}. \tag{14}$$

To upper-bound P_{N-1} in eq. (14), we define a more general quantity P_t by replacing $N-1$ with t in eq. (14). Using the triangle inequality, we have

$$\begin{aligned}
P_t \leq& \alpha(1-\alpha\mu)^t\|\nabla^2_w L_{\mathcal{S}_i}(w^i_t(w_1,\phi_1),\phi_1)) - \nabla^2_w L_{\mathcal{S}_i}(w^i_t(w_2,\phi_2),\phi_2))\| + (1-\alpha\mu)P_{t-1} \\
\leq& (1-\alpha\mu)P_{t-1} + \alpha\rho(1-\alpha\mu)^{\frac{3t}{2}}\|w_1-w_2\| + (1-\alpha\mu)^t\alpha\rho\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|.
\end{aligned} \tag{15}$$

Telescoping eq. (15) over t from 1 to $N-1$ yields

$$P_{N-1} \leq (1-\alpha\mu)^{N-1}P_0 + \sum_{t=1}^{N-1} \alpha\rho(1-\alpha\mu)^{\frac{3t}{2}}\|w_1 - w_2\|(1-\alpha\mu)^{N-1-t} + \sum_{t=1}^{N-1}(1-\alpha\mu)^t\alpha\rho\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|(1-\alpha\mu)^{N-1-t},$$

which, in conjunction with $P_0 \leq \alpha\rho(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|)$, yields

$$\begin{aligned} P_{N-1} \leq & (1-\alpha\mu)^{N-1}\alpha\rho(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|) + \alpha\rho\|w_1 - w_2\|(1-\alpha\mu)^{N-1}\frac{\sqrt{1-\alpha\mu}}{1-\sqrt{1-\alpha\mu}} \\ & + \alpha\rho\left(\frac{2L}{\mu}+1\right)\|\phi_1 - \phi_2\|(N-1)(1-\alpha\mu)^{N-1} \\ \leq & \frac{2\rho}{\mu}(1-\alpha\mu)^{N-1}\|w_1 - w_2\| + \alpha\rho\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|N(1-\alpha\mu)^{N-1}, \end{aligned}$$

which, in conjunction with eq. (14), yields

$$Q \leq \frac{2\rho M}{\mu}(1-\alpha\mu)^{N-1}\|w_1 - w_2\| + \alpha\rho M\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|N(1-\alpha\mu)^{N-1}. \tag{16}$$

Substituting eq. (13) and eq. (16) into eq. (11) yields

$$\begin{aligned} &\Big\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\Big\| \\ &\quad\leq \Big((1-\alpha\mu)^{\frac{3N}{2}}L + \frac{2\rho M}{\mu}(1-\alpha\mu)^{N-1}\Big)\|w_1 - w_2\| \\ &\quad\quad + \Big((1-\alpha\mu)^N L + \alpha\rho M N(1-\alpha\mu)^{N-1}\Big)\left(\frac{2L}{\mu}+1\right)\|\phi_1 - \phi_2\|. \end{aligned} \tag{17}$$

Based on the definition $L^{meta}(w,\phi) = \mathbb{E}_i L_{\mathcal{D}_i}(w_N^i,\phi)$ and using the Jensen's inequality, we have

$$\begin{aligned} &\Big\|\frac{\partial L^{meta}(w,\phi)}{\partial w}\Big|_{(w_1,\phi_1)} - \frac{\partial L^{meta}(w,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\Big\| \\ &\quad \leq \mathbb{E}_i\Big\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\Big\|. \end{aligned} \tag{18}$$

Combining eq. (17) and eq. (18) completes the proof of the first item.

We next prove the Lipschitz property of the partial gradient $\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}$. For notational convenience, we define several quantities below.

$$\begin{aligned} &Q_m(w,\phi) = \nabla_\phi\nabla_w L_{\mathcal{S}_i}(w_m^i(w,\phi),\phi),\ U_m(w,\phi) = \prod_{j=m+1}^{N-1}(I - \alpha\nabla_w^2 L_{\mathcal{S}_i}(w_j^i(w,\phi),\phi)), \\ &V_m(w,\phi) = \nabla_w L_{\mathcal{D}_i}(w_N^i(w,\phi),\phi), \end{aligned} \tag{19}$$

where we let $w_m^i(w,\phi)$ denote the m^{th} inner-loop iteration starting from (w,ϕ). Using an approach similar to the proof for Proposition 1, we have

$$\begin{aligned} \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi} =& -\alpha\sum_{m=0}^{N-1}\nabla_\phi\nabla_w L_{\mathcal{S}_i}(w_m^i,\phi)\prod_{j=m+1}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_j^i,\phi))\nabla_w L_{\mathcal{D}_i}(w_N^i,\phi) \\ & + \nabla_\phi L_{\mathcal{D}_i}(w_N^i,\phi). \end{aligned} \tag{20}$$

Then, we have

$$\begin{aligned} &\Big\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_2,\phi_2)}\Big\| \\ &\leq \alpha\sum_{m=0}^{N-1}\|Q_m(w_1,\phi_1)U_m(w_1,\phi_1)V_m(w_1,\phi_1) - Q_m(w_2,\phi_2)U_m(w_2,\phi_2)V_m(w_2,\phi_2)\| \\ &\quad + \|\nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1) - \nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\|. \end{aligned} \tag{21}$$

Using the triangle inequality, we have

$$\begin{aligned}
&\|Q_m(w_1,\phi_1)U_m(w_1,\phi_1)V_m(w_1,\phi_1) - Q_m(w_2,\phi_2)U_m(w_2,\phi_2)V_m(w_2,\phi_2)\| \\
&\quad\le \underbrace{\|Q_m(w_1,\phi_1) - Q_m(w_2,\phi_2)\|\|U_m(w_1,\phi_1)\|\|V_m(w_1,\phi_1)\|}_{R_1} \\
&\quad\quad + \underbrace{\|Q_m(w_2,\phi_2)\|\|U_m(w_1,\phi_1) - U_m(w_2,\phi_2)\|\|V_m(w_1,\phi_1)\|}_{R_2} \\
&\quad\quad + \underbrace{\|Q_m(w_2,\phi_2)\|\|U_m(w_2,\phi_2)\|\|V_m(w_1,\phi_1) - V_m(w_2,\phi_2)\|}_{R_3}.
\end{aligned} \tag{22}$$

Combining eq. (21) and eq. (22), we have

$$\begin{aligned}
&\Big\| \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_2,\phi_2)}\Big\| \\
&\quad \le \alpha \sum_{m=0}^{N-1}(R_1+R_2+R_3) + \|\nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1) - \nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\|.
\end{aligned} \tag{23}$$

To upper-bound R_1, we have

$$\begin{aligned}
R_1 \le & \tau(\|w_m^i(w_1,\phi_1) - w_m^i(w_2,\phi_2)\| + \|\phi_1-\phi_2\|)(1-\alpha\mu)^{N-m-1}M \\
\le & \tau M(1-\alpha\mu)^{N-\frac{m}{2}-1}\|w_1-w_2\| + \tau M\Big(\frac{2L}{\mu}+1\Big)(1-\alpha\mu)^{N-m-1}\|\phi_1-\phi_2\|,
\end{aligned} \tag{24}$$

where the second inequality follows from Lemma 1.

For R_2, based on Assumptions 1 and 2, we have

$$R_2 \le LM\|U_m(w_1,\phi_1) - U_m(w_2,\phi_2)\|. \tag{25}$$

Using the definitions of $U_m(w_1,\phi_1)$ and $U_m(w_2,\phi_2)$ in eq. (19) and using the triangle inequality, we have

$$\begin{aligned}
&\|U_m(w_1,\phi_1) - U_m(w_2,\phi_2)\| \\
&\quad \le \alpha\|\nabla_w^2 L_{\mathcal{S}_i}(w_{m+1}^i(w_1,\phi_1),\phi_1) - \nabla_w^2 L_{\mathcal{S}_i}(w_{m+1}^i(w_2,\phi_2),\phi_2)\|\|U_{m+1}(w_1,\phi_1)\| \\
&\quad\quad + \|I - \alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{m+1}^i(w_1,\phi_1),\phi_1)\|\|U_{m+1}(w_1,\phi_1) - U_{m+1}(w_2,\phi_2)\| \\
&\quad \le \alpha\rho(1-\alpha\mu)^{N-m-2}(\|w_{m+1}^i(w_1,\phi_1) - w_{m+1}^i(w_2,\phi_2)\| + \|\phi_1-\phi_2\|) \\
&\quad\quad + (1-\alpha\mu)\|U_{m+1}(w_1,\phi_1) - U_{m+1}(w_2,\phi_2)\| \\
&\quad \le \alpha\rho(1-\alpha\mu)^{N-m-2}\Big((1-\alpha\mu)^{\frac{m+1}{2}}\|w_1-w_2\| + \Big(\frac{2L}{\mu}+1\Big)\|\phi_1-\phi_2\|\Big) \\
&\quad\quad\quad + (1-\alpha\mu)\|U_{m+1}(w_1,\phi_1) - U_{m+1}(w_2,\phi_2)\|,
\end{aligned}$$

where the last inequality follows from Lemma 1. Telescoping the above inequality over m yields

$$\begin{aligned}
&\|U_m(w_1,\phi_1) - U_m(w_2,\phi_2)\| \\
&\quad \le (1-\alpha\mu)^{N-m-2}\|U_{N-2}(w_1,\phi_1) - U_{N-2}(w_2,\phi_2)\| \\
&\quad\quad + \sum_{t=0}^{N-m-3}(1-\alpha\mu)^t\alpha\rho(1-\alpha\mu)^{N-m-t-2}\Big((1-\alpha\mu)^{\frac{m+t+1}{2}}\|w_1-w_2\| + \Big(\frac{2L}{\mu}+1\Big)\|\phi_1-\phi_2\|\Big),
\end{aligned}$$

which, in conjunction with eq. (19), yields

$$\begin{aligned}
\|U_m(w_1,\phi_1) - U_m(w_2,\phi_2)\| \le & \left(\frac{\alpha\rho}{1-\alpha\mu} + \frac{2\rho}{\mu}\right)(1-\alpha\mu)^{N-1-\frac{m}{2}}\|w_1-w_2\| \\
& + \alpha(N-1-m)\left(\rho + \frac{2\rho L}{\mu}\right)(1-\alpha\mu)^{N-2-m}\|\phi_1-\phi_2\|.
\end{aligned} \tag{26}$$

Combining eq. (25) and eq. (26) yields

$$R_2 \leq LM\left(\frac{\alpha\rho}{1-\alpha\mu}+\frac{2\rho}{\mu}\right)(1-\alpha\mu)^{N-1-\frac{m}{2}}\|w_1-w_2\| \\ +\alpha LM(N-1-m)\left(\rho+\frac{2\rho L}{\mu}\right)(1-\alpha\mu)^{N-2-m}\|\phi_1-\phi_2\|. \tag{27}$$

For R_3, using the triangle inequality, we have

$$R_3 \leq L(1-\alpha\mu)^{N-m-1}L(\|w_N^i(w_1,\phi_1)-w_N^i(w_2,\phi_2)\|+\|\phi_1-\phi_2\|) \\ \leq L^2(1-\alpha\mu)^{\frac{3N}{2}-m-1}\|w_1-w_2\|+L^2\left(\frac{2L}{\mu}+1\right)(1-\alpha\mu)^{N-1-m}\|\phi_1-\phi_2\|. \tag{28}$$

where the last inequality follows from Lemma 1.

Combine R_1, R_2 and R_3 in eq. (24), eq. (27) and eq. (28), we have

$$\sum_{m=0}^{N-1}(R_1+R_2+R_3) \leq \frac{2\tau M}{\alpha\mu}(1-\alpha\mu)^{\frac{N-1}{2}}\|w_1-w_2\|+\frac{\tau M}{\alpha\mu}\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\| \\ +\frac{2LM}{\alpha\mu}\left(\frac{\alpha\rho}{1-\alpha\mu}+\frac{2\rho}{\mu}\right)(1-\alpha\mu)^{\frac{N-1}{2}}\|w_1-w_2\|+\frac{\alpha LM}{\alpha^2\mu^2}\left(\rho+\frac{2\rho L}{\mu}\right)\|\phi_1-\phi_2\| \\ +\frac{L^2}{\alpha\mu}(1-\alpha\mu)^{\frac{N}{2}}\|w_1-w_2\|+\frac{L^2}{\alpha\mu}\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|. \tag{29}$$

In addition, note that

$$\|\nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1)-\nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\| \\ \leq (1-\alpha\mu)^{\frac{N}{2}}L\|w_1-w_2\|+L\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|. \tag{30}$$

Combining eq. (23), eq. (29), and eq. (30) yields

$$\left\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial\phi}\Big|_{(w_1,\phi_1)}-\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial\phi}\Big|_{(w_2,\phi_2)}\right\| \\ \leq\left(L+\frac{2\tau M}{\mu}+\frac{2LM}{\mu}\left(\frac{\alpha\rho}{1-\alpha\mu}+\frac{2\rho}{\mu}\right)+\frac{L^2}{\mu}\right)(1-\alpha\mu)^{\frac{N-1}{2}}\|w_1-w_2\| \\ +\left(L+\frac{\tau M}{\mu}+\frac{LM\rho}{\mu^2}+\frac{L^2}{\mu}\right)\left(\frac{2L}{\mu}+1\right)\|\phi_1-\phi_2\|, \tag{31}$$

which, using an approach similar to eq. (18), completes the proof.

C.3 Proof of Theorem 1

For notational convenience, we define

$$g_w^i(k)=\frac{\partial L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)}{\partial w_k},\quad g_\phi^i(k)=\frac{\partial L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)}{\partial\phi_k}, \\ L_w=(1-\alpha\mu)^{\frac{3N}{2}}L+\frac{2\rho M}{\mu}(1-\alpha\mu)^{N-1}, L_w'=\Big(L+\alpha\rho MN\Big)(1-\alpha\mu)^{N-1}\left(\frac{2L}{\mu}+1\right), \\ L_\phi=\left(L+\frac{2\tau M}{\mu}+\frac{2LM}{\mu}\left(\frac{\alpha\rho}{1-\alpha\mu}+\frac{2\rho}{\mu}\right)+\frac{L^2}{\mu}\right)(1-\alpha\mu)^{\frac{N-1}{2}}, \\ L_\phi'=\left(L+\frac{\tau M}{\mu}+\frac{LM\rho}{\mu^2}+\frac{L^2}{\mu}\right)\left(\frac{2L}{\mu}+1\right). \tag{32}$$

Then, the updates of Algorithm 1 are given by

$$w_{k+1}=w_k-\frac{\beta_w}{B}\sum_{i\in\mathcal{B}_k}g_w^i(k)\ \text{ and }\ \phi_{k+1}=\phi_k-\frac{\beta_\phi}{B}\sum_{i\in\mathcal{B}_k}g_\phi^i(k). \tag{33}$$

Based on the smoothness properties established in eq. (17) and eq. (31) in the proof of Proposition 2, we have

$$L^{meta}(w_{k+1},\phi_k) \leq L^{meta}(w_k,\phi_k) + \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}, w_{k+1}-w_k \right\rangle + \frac{L_w}{2}\|w_{k+1}-w_k\|^2,$$

$$L^{meta}(w_{k+1},\phi_{k+1}) \leq L^{meta}(w_{k+1},\phi_k) + \left\langle \frac{\partial L^{meta}(w_{k+1},\phi_k)}{\partial \phi_k}, \phi_{k+1}-\phi_k \right\rangle + \frac{L'_\phi}{2}\|\phi_{k+1}-\phi_k\|^2.$$

Adding the above two inequalities, we have

$$\begin{aligned} L^{meta}(w_{k+1},\phi_{k+1}) \leq & L^{meta}(w_k,\phi_k) + \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}, w_{k+1}-w_k \right\rangle + \frac{L_w}{2}\|w_{k+1}-w_k\|^2 \\ & + \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}, \phi_{k+1}-\phi_k \right\rangle + \frac{L'_\phi}{2}\|\phi_{k+1}-\phi_k\|^2 \\ & + \left\langle \frac{\partial L^{meta}(w_{k+1},\phi_k)}{\partial \phi_k} - \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}, \phi_{k+1}-\phi_k \right\rangle. \end{aligned} \tag{34}$$

Based on the Cauchy-Schwarz inequality, we have

$$\begin{aligned} \Big\langle \frac{\partial L^{meta}(w_{k+1},\phi_k)}{\partial \phi_k} - & \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}, \phi_{k+1}-\phi_k \Big\rangle \\ & \leq L_\phi \|w_{k+1}-w_k\|\|\phi_{k+1}-\phi_k\| \\ & \leq \frac{L_\phi}{2}\|w_{k+1}-w_k\|^2 + \frac{L_\phi}{2}\|\phi_{k+1}-\phi_k\|^2. \end{aligned} \tag{35}$$

Combining eq. (34) and eq. (35), we have

$$\begin{aligned} L^{meta}(w_{k+1},\phi_{k+1}) \leq & L^{meta}(w_k,\phi_k) + \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}, w_{k+1}-w_k \right\rangle + \frac{L_w + L_\phi}{2}\|w_{k+1}-w_k\|^2 \\ & + \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}, \phi_{k+1}-\phi_k \right\rangle + \frac{L_\phi + L'_\phi}{2}\|\phi_{k+1}-\phi_k\|^2, \end{aligned}$$

which, in conjunction with the updates in eq. (33), yields

$$\begin{aligned} & L^{meta}(w_{k+1},\phi_{k+1}) \\ & \quad \leq L^{meta}(w_k,\phi_k) - \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}, \frac{\beta_w}{B}\sum_{i\in\mathcal{B}_k} g_w^i(k) \right\rangle + \frac{L_w + L_\phi}{2}\left\| \frac{\beta_w}{B}\sum_{i\in\mathcal{B}_k} g_w^i(k)\right\|^2 \\ & \quad - \left\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}, \frac{\beta_\phi}{B}\sum_{i\in\mathcal{B}_k} g_\phi^i(k) \right\rangle + \frac{L_\phi + L'_\phi}{2}\left\| \frac{\beta_\phi}{B}\sum_{i\in\mathcal{B}_k} g_\phi^i(k)\right\|^2. \end{aligned} \tag{36}$$

Let $\mathbb{E}_k = \mathbb{E}(\cdot|w_k,\phi_k)$. Then, conditioning on w_k,ϕ_k, and taking expectation over eq. (36), we have

$$\begin{aligned} \mathbb{E}_k L^{meta}(w_{k+1},\phi_{k+1}) \overset{(i)}{\leq} & L^{meta}(w_k,\phi_k) - \beta_w \left\| \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}\right\|^2 + \frac{L_w + L_\phi}{2}\mathbb{E}_k\left\| \frac{\beta_w}{B}\sum_{i\in\mathcal{B}_k} g_w^i(k)\right\|^2 \\ & - \beta_\phi \left\| \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}\right\| + \frac{L_\phi + L'_\phi}{2}\mathbb{E}_k\left\| \frac{\beta_\phi}{B}\sum_{i\in\mathcal{B}_k} g_\phi^i(k)\right\|^2 \\ \leq & L^{meta}(w_k,\phi_k) - \beta_w \left\| \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}\right\|^2 + \frac{(L_w + L_\phi)\beta_w^2}{2B}\mathbb{E}_k\left\|g_w^i(k)\right\|^2 \\ & + \frac{L_\phi + L_w}{2}\beta_w^2 \left\| \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}\right\|^2 - \beta_\phi \left\| \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}\right\|^2 \\ & + \frac{L_\phi + L'_\phi}{2}\left(\frac{\beta_\phi^2}{B}\mathbb{E}_k\left\|g_\phi^i(k)\right\|^2 + \beta_\phi^2 \left\| \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}\right\|^2 \right), \end{aligned} \tag{37}$$

where (i) follows from the fact that $\mathbb{E}_k g_w^i(k) = \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}$ and $\mathbb{E}_k g_\phi^i(k) = \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}$.

Our next step is to upper-bound $\mathbb{E}_k\|g_w^i(k)\|^2$ and $\mathbb{E}_k\|g_\phi^i(k)\|^2$ in eq. (37). Based on the definitions of $g_w^i(k)$ in eq. (32) and using the explicit forms of the meta gradients in Proposition 1, we have

$$\begin{aligned}\mathbb{E}_k\|g_w^i(k)\|^2 \leq & \mathbb{E}_k\Big\|\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k))\nabla_w L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)\Big\|^2 \\ \leq & (1-\alpha\mu)^{2N}M^2. \end{aligned} \tag{38}$$

Using an approach similar to eq. (38), we have

$$\begin{aligned}\mathbb{E}_k\|g_\phi^i(k)\|^2 \leq & 2\mathbb{E}_k\Big\|\alpha\sum_{m=0}^{N-1}\nabla_\phi\nabla_w L_{\mathcal{S}_i}(w_{k,m}^i,\phi_k)\prod_{j=m+1}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{k,j}^i,\phi_k))\nabla_w L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)\Big\|^2 \\ & + 2\|\nabla_\phi L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)\|^2 \\ \leq & 2\alpha^2 L^2 M^2\mathbb{E}_k\Big(\sum_{m=0}^{N-1}(1-\alpha\mu)^{N-1-m}\Big)^2 + 2M^2 \\ < & \frac{2L^2M^2}{\mu^2} + 2M^2 < 2M^2\left(\frac{L^2}{\mu^2}+1\right). \end{aligned} \tag{39}$$

Substituting eq. (38) and eq. (39) into eq. (37) yields

$$\begin{aligned}\mathbb{E}_k L^{meta}(w_{k+1},\phi_{k+1}) \leq & L^{meta}(w_k,\phi_k) - \left(\beta_w - \frac{L_w+L_\phi}{2}\beta_w^2\right)\left\|\frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}\right\|^2 \\ & + \frac{(L_w+L_\phi)\beta_w^2}{2B}(1-\alpha\mu)^{2N}M^2 - \left(\beta_\phi - \frac{L_\phi+L_\phi'}{2}\beta_\phi^2\right)\left\|\frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}\right\|^2 \\ & + \frac{(L_\phi+L_\phi')\beta_\phi^2}{B}M^2\left(\frac{L^2}{\mu^2}+1\right). \end{aligned} \tag{40}$$

Let $\beta_w = \frac{1}{L_w+L_\phi}$ and $\beta_\phi = \frac{1}{L_\phi+L_\phi'}$. Then, unconditioning on w_k and ϕ_k and telescoping eq. (40) over k from 0 to $K-1$ yield

$$\begin{aligned}&\frac{\beta_w}{2}\frac{1}{K}\sum_{k=0}^{K-1}\mathbb{E}\left\|\frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}\right\|^2 + \frac{\beta_\phi}{2}\frac{1}{K}\sum_{k=0}^{K-1}\mathbb{E}\left\|\frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}\right\|^2 \\ &\quad\leq \frac{L^{meta}(w_0,\phi_0)-\min_{w,\phi}L^{meta}(w,\phi)}{K} + \frac{\beta_w}{2B}(1-\alpha\mu)^{2N}M^2 + \frac{\beta_\phi}{B}M^2\left(\frac{L^2}{\mu^2}+1\right). \end{aligned} \tag{41}$$

Let $\Delta = L^{meta}(w_0,\phi_0)-\min_{w,\phi}L^{meta}(w,\phi)$ and let ξ be chosen from $\{0,...,K-1\}$ uniformly at random. Then, we have

$$\begin{aligned}\mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi,\phi_\xi)}{\partial w_\xi}\right\|^2 &\leq \frac{2\Delta(L_w+L_\phi)}{K} + \frac{(1-\alpha\mu)^{2N}M^2}{B} + \frac{L_w+L_\phi}{L_\phi+L_\phi'}\frac{2}{B}M^2\left(\frac{L^2}{\mu^2}+1\right), \\ \mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi,\phi_\xi)}{\partial \phi_\xi}\right\|^2 &\leq \frac{2\Delta(L_\phi+L_\phi')}{K} + \frac{L_\phi+L_\phi'}{L_w+L_\phi}\frac{1}{B}(1-\alpha\mu)^{2N}M^2 + \frac{2}{B}M^2\left(\frac{L^2}{\mu^2}+1\right), \end{aligned}$$

which, in conjunction with the definitions of L_ϕ, L_ϕ' and L_w in eq. (32) and $\alpha = \frac{\mu}{L^2}$, yields

$$\begin{aligned}\mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi,\phi_\xi)}{\partial w_\xi}\right\|^2 &\leq \mathcal{O}\left(\frac{\frac{1}{\mu^2}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{N}{2}}}{K} + \frac{\frac{1}{\mu}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{N}{2}}}{B}\right), \\ \mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi,\phi_\xi)}{\partial \phi_\xi}\right\|^2 &\leq \mathcal{O}\left(\frac{\frac{1}{\mu^2}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{N}{2}}+\frac{1}{\mu^3}}{K} + \frac{\frac{1}{\mu}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{3N}{2}}+\frac{1}{\mu^2}}{B}\right). \end{aligned}$$

To achieve an ϵ-stationary point, i.e., $\mathbb{E}\left\|\frac{\partial L^{meta}(w,\phi)}{\partial w}\right\|^2 < \epsilon, \mathbb{E}\left\|\frac{\partial L^{meta}(w,\phi)}{\partial w}\right\|^2 < \epsilon$, ANIL requires at most

$$\begin{aligned}
KBN =&\mathcal{O}\left(\frac{L^2}{\mu^2}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{N}{2}}+\frac{L^3}{\mu^3}\right)\left(\frac{L}{\mu}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{3N}{2}}+\frac{L^2}{\mu^2}\right)N\epsilon^{-2} \\
\leq&\mathcal{O}\left(\frac{N}{\mu^4}\left(1-\frac{\mu^2}{L^2}\right)^{\frac{N}{2}}+\frac{N}{\mu^5}\right)\epsilon^{-2}
\end{aligned}$$

gradient evaluations in w, $KB = \mathcal{O}\Big(\mu^{-4}\left(1-\frac{\mu^2}{L^2}\right)^{N/2}+\mu^{-5}\Big)\epsilon^{-2}$ gradient evaluations in ϕ, and $KBN = \mathcal{O}\Big(\frac{N}{\mu^4}\left(1-\frac{\mu^2}{L^2}\right)^{N/2}+\frac{N}{\mu^5}\Big)\epsilon^{-2}$ evaluations of second-order derivatives.

D Proof in Section 3.2: Nonconvex Inner Loop

D.1 Proof of Proposition 3

Based on the explicit forms of the meta gradient in eq. (10) and using an approach similar to eq. (11), we have

$$\begin{aligned}
&\left\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_1,\phi_1)}-\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\right\| \\
&=\Big\|\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1))\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1) \\
&\qquad-\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2))\nabla_w L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\Big\|, \qquad (42)
\end{aligned}$$

where $w_m^i(w,\phi)$ is obtained through the gradient descent steps in eq. (12).

Using the triangle inequality in eq. (42) yields

$$\begin{aligned}
&\left\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_1,\phi_1)}-\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\right\| \\
\leq&\Big\|\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2))\Big\|\Big\|\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\Big\| \\
&+\Big\|\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1))\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1) \\
&\qquad-\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2))\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1)\Big\|. \qquad (43)
\end{aligned}$$

Our next two steps are to upper-bound the two terms at the right hand side of eq. (43), respectively.

Step 1: Upper-bound the first term at the right hand side of eq. (43).

$$\begin{aligned}
&\Big\|\prod_{m=0}^{N-1}(I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2))\Big\|\Big\|\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\Big\| \\
&\overset{(i)}{\leq}(1+\alpha L)^N\Big\|\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\Big\| \\
&\overset{(ii)}{\leq}(1+\alpha L)^N L(\|w_N^i(w_1,\phi_1)-w_N^i(w_2,\phi_2)\|+\|\phi_1-\phi_2\|), \qquad (44)
\end{aligned}$$

where (i) follows from the fact that $\|\nabla_w^2 L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2)\|\leq L$, and (ii) follows from Assumption 1. Based on the gradient descent steps in eq. (12), we have, for any $0\leq m\leq N-1$,

$$\begin{aligned}
&w_{m+1}^i(w_1,\phi_1)-w_{m+1}^i(w_2,\phi_2) \\
&\quad=w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2)-\alpha\big(\nabla_w L_{\mathcal{S}_i}(w_m^i(w_1,\phi_1),\phi_1)-\nabla_w L_{\mathcal{S}_i}(w_m^i(w_2,\phi_2),\phi_2)\big).
\end{aligned}$$

Based on the above equality, we further obtain

$$\begin{aligned}\|w^i_{m+1}(w_1,\phi_1) - w^i_{m+1}(w_2,\phi_2)\| \leq & \|w^i_m(w_1,\phi_1) - w^i_m(w_2,\phi_2)\| \\ & + \alpha\|\nabla_w L_{\mathcal{S}_i}(w^i_m(w_1,\phi_1),\phi_1) - \nabla_w L_{\mathcal{S}_i}(w^i_m(w_2,\phi_2),\phi_2)\| \\ \leq & (1+\alpha L)\|w^i_m(w_1,\phi_1) - w^i_m(w_2,\phi_2)\| + \alpha L\|\phi_1 - \phi_2\|,\end{aligned}$$

where the last inequality follows from Assumption 1. Telescoping the above inequality over m from 0 to $N-1$ yields

$$\|w^i_N(w_1,\phi_1) - w^i_N(w_2,\phi_2)\| \leq (1+\alpha L)^N\|w_1 - w_2\| + ((1+\alpha L)^N - 1)\|\phi_1 - \phi_2\|. \tag{45}$$

Combining eq. (44) and eq. (45) yields

$$\begin{aligned}&\Big\|\prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_2,\phi_2),\phi_2))\Big\|\Big\|\nabla_w L_{\mathcal{D}_i}(w^i_N(w_1,\phi_1),\phi_1) - \nabla_w L_{\mathcal{D}_i}(w^i_N(w_2,\phi_2),\phi_2)\Big\| \\ &\leq (1+\alpha L)^{2N} L(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|). \end{aligned}\tag{46}$$

Step 2: Upper-bound the second term at the right hand side of eq. (43).

Based on item 2 in Assumption 1, we have that $\|\nabla_w L_{\mathcal{D}_i}(\cdot,\cdot)\| \leq M$. Then, the second term at the right hand side of eq. (43) is further upper-bounded by

$$M\underbrace{\Big\|\prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_1,\phi_1),\phi_1)) - \prod_{m=0}^{N-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m(w_2,\phi_2),\phi_2))\Big\|}_{P_{N-1}}. \tag{47}$$

In order to upper-bound P_{N-1} in eq. (47), we define a more general quantity P_t by replacing $N-1$ with t in eq. (47). Based on the triangle inequality, we have

$$\begin{aligned}P_t \leq & \alpha\Big\|\prod_{m=0}^{t-1}(I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_m,\phi_1))\Big\|\Big\|\nabla^2_w L_{\mathcal{S}_i}(w^i_t(w_1,\phi_1),\phi_1) - \nabla^2_w L_{\mathcal{S}_i}(w^i_t(w_2,\phi_2),\phi_2)\Big\| \\ & + P_{t-1}\Big\|I - \alpha\nabla^2_w L_{\mathcal{S}_i}(w^i_t(w_2,\phi_2),\phi_2)\Big\| \\ \leq & \alpha(1+\alpha L)^t\rho(\|w^i_t(w_1,\phi_1) - w^i_t(w_2,\phi_2)\| + \|\phi_1 - \phi_2\|) + (1+\alpha L)P_{t-1} \\ \overset{(i)}{\leq} & \alpha\rho(1+\alpha L)^{2t}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|) + (1+\alpha L)P_{t-1},\end{aligned}$$

where (i) follows from eq. (45). Rearranging the above inequality, we have

$$\begin{aligned}P_t - & \frac{\rho}{L}(1+\alpha L)^{2t+1}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|) \\ & \leq (1+\alpha L)(P_{t-1} - \frac{\rho}{L}(1+\alpha L)^{2t-1}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|)).\end{aligned}\tag{48}$$

Telescoping eq. (48) over t from 1 to $N-1$ yields

$$\begin{aligned}P_{N-1} - & \frac{\rho}{L}(1+\alpha L)^{2N-1}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|) \\ & \leq (1+\alpha L)^N\Big(P_0 - \frac{\rho}{L}(1+\alpha L)(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|)\Big),\end{aligned}$$

which, in conjunction with $P_0 = \alpha\|\nabla^2_w L_{\mathcal{S}_i}(w_1,\phi_1) - \nabla^2_w L_{\mathcal{S}_i}(w_2,\phi_2)\| \leq \alpha\rho(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|)$, yields

$$\begin{aligned}P_{N-1} - & \frac{\rho}{L}(1+\alpha L)^{2N-1}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|) \\ & \leq (1+\alpha L)^N\Big(\frac{\rho}{L}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|)\Big) \\ & \leq \frac{\rho}{L}(1+\alpha L)^{2N-1}(\|w_1 - w_2\| + \|\phi_1 - \phi_2\|),\end{aligned}\tag{49}$$

where the last inequality follows because $N \geq 1$. Combining eq. (47), and eq. (49), we have that the second term at the right hand side of eq. (43) is upper-bounded by

$$\frac{2M\rho}{L}(1+\alpha L)^{2N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|). \tag{50}$$

Step 3: Combine two bounds in Steps 1 and 2.

Combining eq. (46), eq. (50), and using $\alpha < \mathcal{O}(\frac{1}{N})$, we have

$$\begin{aligned}
&\Big\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial w}\Big|_{(w_2,\phi_2)}\Big\| \\
&\quad\leq \Big(1+\alpha L+\frac{2M\rho}{L}\Big)(1+\alpha L)^{2N-1}L(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\
&\quad\leq \text{poly}(M,\rho,\alpha,L)N(\|w_1-w_2\|+\|\phi_1-\phi_2\|),
\end{aligned} \tag{51}$$

which, using an approach similar to eq. (18), completes the proof of the first item in Proposition 3.

We next prove the Lipschitz property of the partial gradient $\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}$. Using an approach similar to eq. (21) and eq. (22), we have

$$\begin{aligned}
&\Big\|\frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_2,\phi_2)}\Big\| \\
&\leq \alpha\sum_{m=0}^{N-1}(R_1+R_2+R_3) + \|\nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1) - \nabla_\phi L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\|,
\end{aligned} \tag{52}$$

where R_1, R_2 and R_3 are defined in eq. (22).

To upper-bound R_1 in the above inequality, we have

$$\begin{aligned}
R_1 &\overset{(i)}{\leq} \tau(\|w_m^i(w_1,\phi_1)-w_m^i(w_2,\phi_2)\|+\|\phi_1-\phi_2\|)(1+\alpha L)^{N-m-1}M \\
&\overset{(ii)}{\leq} \tau M(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|),
\end{aligned} \tag{53}$$

where (i) follows from Assumptions 1 and 2 and (ii) follows from eq. (45).

For R_2, using the triangle inequality, we have

$$\begin{aligned}
&\|U_m(w_1,\phi_1)-U_m(w_2,\phi_2)\| \\
&\quad\leq \alpha\|\nabla_w^2 L_{\mathcal{S}_i}(w_{m+1}^i(w_1,\phi_1),\phi_1) - \nabla_w^2 L_{\mathcal{S}_i}(w_{m+1}^i(w_2,\phi_2),\phi_2)\|\|U_{m+1}(w_1,\phi_1)\| \\
&\qquad + \|I-\alpha\nabla_w^2 L_{\mathcal{S}_i}(w_{m+1}^i(w_1,\phi_1),\phi_1)\|\|U_{m+1}(w_1,\phi_1)-U_{m+1}(w_2,\phi_2)\| \\
&\quad\leq \alpha\rho(1+\alpha L)^{N-m-2}(\|w_{m+1}^i(w_1,\phi_1)-w_{m+1}^i(w_2,\phi_2)\|+\|\phi_1-\phi_2\|) \\
&\qquad + (1+\alpha L)\|U_{m+1}(w_1,\phi_1)-U_{m+1}(w_2,\phi_2)\| \\
&\quad\leq \alpha\rho(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\
&\qquad + (1+\alpha L)\|U_{m+1}(w_1,\phi_1)-U_{m+1}(w_2,\phi_2)\|.
\end{aligned} \tag{54}$$

Telescoping the above inequality over m yields

$$\begin{aligned}
&\|U_m(w_1,\phi_1)-U_m(w_2,\phi_2)\| + \frac{\rho}{L}(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\
&\leq (1+\alpha L)^{N-m-2}\Big(\|U_{N-2}(w_1,\phi_1)-U_{N-2}(w_2,\phi_2)\| + \frac{\rho}{L}(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|)\Big),
\end{aligned}$$

which, in conjunction with

$$\begin{aligned}
\|U_{N-2}(w_1,\phi_1)-U_{N-2}(w_2,\phi_2)\| &= \alpha\|\nabla_w^2 L_{\mathcal{S}_i}(w_{N-1}^i(w_1,\phi_1),\phi_1) - \nabla_w^2 L_{\mathcal{S}_i}(w_{N-1}^i(w_2,\phi_2),\phi_2)\| \\
&\leq \alpha\rho(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|),
\end{aligned}$$

yields that

$$\begin{aligned}
\|U_m(w_1,\phi_1)-U_m(w_2,\phi_2)\| \leq& \Big(\alpha\rho+\frac{\rho}{L}\Big)(1+\alpha L)^{2N-m-3}(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\
&- \frac{\rho}{L}(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|).
\end{aligned} \tag{55}$$

Based on Assumption 1, we have $\|Q_m(w_2,\phi_2)\| \le L$ and $\|V_m(w_1,\phi_1)\| \le M$, which, combined with eq. (55) and the definition of R_2 in eq. (22), yields

$$\begin{aligned} R_2 \le & ML\Big(\alpha\rho + \frac{\rho}{L}\Big)(1+\alpha L)^{2N-m-3}(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\ & - M\rho(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|). \end{aligned} \tag{56}$$

For R_3, using Assumption 1, we have

$$\begin{aligned} R_3 \le & L(1+\alpha L)^{N-m-1}\|\nabla_w L_{\mathcal{D}_i}(w_N^i(w_1,\phi_1),\phi_1) - \nabla_w L_{\mathcal{D}_i}(w_N^i(w_2,\phi_2),\phi_2)\| \\ \le & L^2(1+\alpha L)^{2N-m-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|), \end{aligned} \tag{57}$$

where the last inequality follows from eq. (45). Combining eq. (53), eq. (56) and eq. (57) yields

$$\begin{aligned} R_1 + R_2 + R_3 \le & M(\tau-\rho)(1+\alpha L)^{N-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\ & + M\rho(1+\alpha L)^{2N-m-2}(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\ & + L^2(1+\alpha L)^{2N-m-1}(\|w_1-w_2\|+\|\phi_1-\phi_2\|). \end{aligned} \tag{58}$$

Combining eq. (52), eq. (58), and using eq. (45) and $\alpha < \mathcal{O}(\frac{1}{N})$, we have

$$\begin{aligned} & \Big\| \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_1,\phi_1)} - \frac{\partial L_{\mathcal{D}_i}(w_N^i,\phi)}{\partial \phi}\Big|_{(w_2,\phi_2)}\Big\| \\ & \le \Big(\alpha M(\tau-\rho)N(1+\alpha L)^{N-1} + \Big(L + \frac{\rho M}{L}\Big)(1+\alpha L)^{2N}\Big)(\|w_1-w_2\|+\|\phi_1-\phi_2\|) \\ & \le \text{poly}(M,\rho,\tau,\alpha,L)N(\|w_1-w_2\|+\|\phi_1-\phi_2\|), \end{aligned} \tag{59}$$

which, using an approach similar to eq. (18), finishes the proof of the second item in Proposition 3.

D.2 Proof of Theorem 2

For notational convenience, we define

$$\begin{aligned} & g_w^i(k) = \frac{\partial L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)}{\partial w_k}, \quad g_\phi^i(k) = \frac{\partial L_{\mathcal{D}_i}(w_{k,N}^i,\phi_k)}{\partial \phi_k}, \\ & L_w = \big(L + \alpha L^2 + 2M\rho\big)(1+\alpha L)^{2N-1}, \\ & L_\phi = \alpha M(\tau-\rho)N(1+\alpha L)^{N-1} + \left(L + \frac{\rho M}{L}\right)(1+\alpha L)^{2N}. \end{aligned} \tag{60}$$

Based on the smoothness properties established in eq. (51) and eq. (59) in the proof of Proposition 3, we have

$$\begin{aligned} L^{meta}(w_{k+1},\phi_k) \le & L^{meta}(w_k,\phi_k) + \Big\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}, w_{k+1}-w_k\Big\rangle + \frac{L_w}{2}\|w_{k+1}-w_k\|^2, \\ L^{meta}(w_{k+1},\phi_{k+1}) \le & L^{meta}(w_{k+1},\phi_k) + \Big\langle \frac{\partial L^{meta}(w_{k+1},\phi_k)}{\partial \phi_k}, \phi_{k+1}-\phi_k\Big\rangle + \frac{L_\phi}{2}\|\phi_{k+1}-\phi_k\|^2. \end{aligned}$$

Adding the above two inequalities, and using an approach similar to eq. (36), we have

$$\begin{aligned} & L^{meta}(w_{k+1},\phi_{k+1}) \\ & \le L^{meta}(w_k,\phi_k) - \Big\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial w_k}, \frac{\beta_w}{B}\sum_{i\in\mathcal{B}_k} g_w^i(k)\Big\rangle + \frac{L_w+L_\phi}{2}\Big\|\frac{\beta_w}{B}\sum_{i\in\mathcal{B}_k} g_w^i(k)\Big\|^2 \\ & \quad - \Big\langle \frac{\partial L^{meta}(w_k,\phi_k)}{\partial \phi_k}, \frac{\beta_\phi}{B}\sum_{i\in\mathcal{B}_k} g_\phi^i(k)\Big\rangle + L_\phi\Big\|\frac{\beta_\phi}{B}\sum_{i\in\mathcal{B}_k} g_\phi^i(k)\Big\|^2. \end{aligned} \tag{61}$$

Let $\mathbb{E}_k = \mathbb{E}(\cdot|w_k, \phi_k)$. Then, conditioning on w_k, ϕ_k, taking expectation over eq. (61) and using an approach similar to eq. (37), we have

$$\begin{aligned}\mathbb{E}_k L^{meta}(w_{k+1}, \phi_{k+1}) \leq & L^{meta}(w_k, \phi_k) - \beta_w \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial w_k} \right\|^2 + \frac{(L_w + L_\phi)\beta_w^2}{2B} \mathbb{E}_k \left\| g_w^i(k) \right\|^2 \\ & + \frac{L_\phi + L_w}{2} \beta_w^2 \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial w_k} \right\|^2 - \beta_\phi \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial \phi_k} \right\|^2 \\ & + L_\phi \left(\frac{\beta_\phi^2}{B} \mathbb{E}_k \left\| g_\phi^i(k) \right\|^2 + \beta_\phi^2 \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial \phi_k} \right\|^2 \right). \end{aligned} \tag{62}$$

Our next step is to upper-bound $\mathbb{E}_k \left\| g_w^i(k) \right\|^2$ and $\mathbb{E}_k \left\| g_\phi^i(k) \right\|^2$ in eq. (62). Based on the definitions of $g_w^i(k)$ in eq. (60) and Proposition 1, we have

$$\begin{aligned}\mathbb{E}_k \left\| g_w^i(k) \right\|^2 \leq & \mathbb{E}_k \left\| \frac{\partial L_{\mathcal{D}_i}(w_{k,N}^i, \phi_k)}{\partial w_k} \right\|^2 = \mathbb{E}_k \left\| \prod_{m=0}^{N-1} (I - \alpha \nabla_w^2 L_{\mathcal{S}_i}(w_{k,m}^i, \phi_k)) \nabla_w L_{\mathcal{D}_i}(w_{k,N}^i, \phi_k) \right\|^2 \\ \leq & \mathbb{E}_k (1 + \alpha L)^{2N} M^2 = (1 + \alpha L)^{2N} M^2. \end{aligned} \tag{63}$$

Using an approach similar to eq. (63), we have

$$\begin{aligned}\mathbb{E}_k \left\| g_\phi^i(k) \right\|^2 \leq & 2\mathbb{E}_k \left\| \alpha \sum_{m=0}^{N-1} \nabla_\phi \nabla_w L_{\mathcal{S}_i}(w_{k,m}^i, \phi_k) \prod_{j=m+1}^{N-1} (I - \alpha \nabla_w^2 L_{\mathcal{S}_i}(w_{k,j}^i, \phi_k)) \nabla_w L_{\mathcal{D}_i}(w_{k,N}^i, \phi_k) \right\|^2 \\ & + 2 \| \nabla_\phi L_{\mathcal{D}_i}(w_{k,N}^i, \phi_k) \|^2 \\ \leq & 2\alpha^2 L^2 M^2 \mathbb{E}_k \Big(\sum_{m=0}^{N-1} (1 + \alpha L)^{N-1-m} \Big)^2 + 2M^2 \\ < & 2M^2 (1 + \alpha L)^N - 1)^2 + 2M^2 < 2M^2 (1 + \alpha L)^{2N}. \end{aligned} \tag{64}$$

Substituting eq. (63) and eq. (64) into eq. (62), we have

$$\begin{aligned}\mathbb{E}_k L^{meta}(w_{k+1}, \phi_{k+1}) \leq & L^{meta}(w_k, \phi_k) - \left(\beta_w - \frac{L_w + L_\phi}{2} \beta_w^2 \right) \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial w_k} \right\|^2 \\ & + \frac{(L_w + L_\phi)\beta_w^2}{2B} (1 + \alpha L)^{2N} M^2 - \left(\beta_\phi - L_\phi \beta_\phi^2 \right) \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial \phi_k} \right\|^2 \\ & + \frac{2L_\phi \beta_\phi^2}{B} (1 + \alpha L)^{2N} M^2. \end{aligned} \tag{65}$$

Set $\beta_w = \frac{1}{L_w + L_\phi}$ and $\beta_\phi = \frac{1}{2L_\phi}$. Then, unconditioning on w_k, ϕ_k in eq. (65), we have

$$\begin{aligned}\mathbb{E} L^{meta}(w_{k+1}, \phi_{k+1}) \leq & \mathbb{E} L^{meta}(w_k, \phi_k) - \frac{\beta_w}{2} \mathbb{E} \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial w_k} \right\|^2 + \frac{\beta_w}{2B} (1 + \alpha L)^{2N} M^2 \\ & - \frac{\beta_\phi}{2} \mathbb{E} \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial \phi_k} \right\|^2 + \frac{\beta_\phi}{B} (1 + \alpha L)^{2N} M^2. \end{aligned}$$

Telescoping the above equality over k from 0 to $K - 1$ yields

$$\begin{aligned} & \frac{\beta_w}{2} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E} \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial w_k} \right\|^2 + \frac{\beta_\phi}{2} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E} \left\| \frac{\partial L^{meta}(w_k, \phi_k)}{\partial \phi_k} \right\|^2 \\ & \quad \leq \frac{L^{meta}(w_0, \phi_0) - \min_{w,\phi} L^{meta}(w, \phi)}{K} + \frac{\beta_w + 2\beta_\phi}{2B} (1 + \alpha L)^{2N} M^2. \end{aligned} \tag{66}$$

Let $\Delta = L^{meta}(w_0, \phi_0) - \min_{w,\phi} L^{meta}(w, \phi) > 0$ and let ξ be chosen from $\{0, ..., K-1\}$ uniformly at random. Then, eq. (66) further yields

$$\begin{aligned}
\mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi, \phi_\xi)}{\partial w_\xi}\right\|^2 &\leq \frac{2\Delta(L_w + L_\phi)}{K} + \frac{1 + \frac{L_w + L_\phi}{L_\phi}}{B}(1+\alpha L)^{2N} M^2 \\
\mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi, \phi_\xi)}{\partial \phi_\xi}\right\|^2 &\leq \frac{4\Delta L_\phi}{K} + \frac{2 + \frac{2L_\phi}{L_w + L_\phi}}{B}(1+\alpha L)^{2N} M^2,
\end{aligned}$$

which, in conjunction with the definitions of L_w and L_ϕ in eq. (60) and using $\alpha < \mathcal{O}(\frac{1}{N})$, yields

$$\begin{aligned}
\mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi, \phi_\xi)}{\partial w_\xi}\right\|^2 &\leq \mathcal{O}\left(\frac{N}{K} + \frac{N}{B}\right), \\
\mathbb{E}\left\|\frac{\partial L^{meta}(w_\xi, \phi_\xi)}{\partial \phi_\xi}\right\|^2 &\leq \mathcal{O}\left(\frac{N}{K} + \frac{N}{B}\right). \qquad (67)
\end{aligned}$$

To achieve an ϵ-stationary point, i.e., $\mathbb{E}\left\|\frac{\partial L^{meta}(w,\phi)}{\partial w}\right\|^2 < \epsilon, \mathbb{E}\left\|\frac{\partial L^{meta}(w,\phi)}{\partial w}\right\|^2 < \epsilon$, K and B need to be at most $\mathcal{O}(N\epsilon^{-2})$, which, in conjunction with the gradient forms in Proposition 1, completes the complexity results.