

1 We would like to thank the reviewers for their interesting comments and questions.

2 **Reviewer 1:** It is great to hear that you really like the conceptual point of our paper and that you appreciate our strong
3 technical contribution.

4 **Q1:** *Does chaotic behavior persist if agents use different values of epsilon?* Great question! In Appendix I, pages
5 33-36 of the full paper, we examine the case of two learning rates ϵ_1 and ϵ_2 with both theoretical and experimental
6 results (see Figs 11, 12). These results can be extended to any finite number of learning rates by paying mostly a
7 notational overhead. Rev. 1 has an excellent idea about a continuum of learning rates distributed (without atoms) over
8 the population. This is a very interesting open question. Novel ideas and techniques would be required. Unfortunately,
9 this setting is not really conducive to simulations as the full system state is a continuous measure. A discretized version
10 with a finite large set of learning rates would result in a more complex version of our ϵ_1 and ϵ_2 figures. We will be
11 happy to expand upon this in a discussion section for future work.

12 **Q2:** *Scaling of costs in $[0, 1]$: "If an agent is repeatedly interacting with a system at a fixed load, I'd expect them to*
13 *adapt their learning to that environment, which (arguably) corresponds to scaling to $[0, 1]$."* There is a subtle but
14 important distinction to be made here. The normalization you describe here does not result in games with costs in $[0, 1]$,
15 not even approximately. Let's assume when you travel to work it takes somewhere between 10 to 30 minutes and hence
16 you normalize your costs by dividing by 30 or even 60 to be safe. Your worst case cost can still be arbitrarily larger
17 than 1 especially in games with large populations as it corresponds to the case where you choose the longest possible
18 route and *everyone else* in your city chooses that route too. If one was to work on a formal model of the suggested
19 normalization two things should be clear: 1) whether chaos emerges or not still has to be carefully explored using our
20 techniques 2) this is not the normalization in PoA literature that assumes that not just *your typical operating costs* but
21 the much more demanding condition that even *worst case costs* lie in $[0, 1]$. "*if... the load of a system is actually*
22 *changing (slowly) over time, I think it's reasonable that agents wouldn't update their scaling in response."* We agree
23 and we want to point out that in (Lykouris et al. Learning and efficiency in games with dynamic population, SODA
24 2016) although the authors explicitly focus on a time-dependent population size, they assume that *all* stage game (worst
25 case) costs are bounded in $[0, 1]$ (e.g. Theorem 3.1). Since this modelling assumption is not always easily applicable
26 and as we show it can totally dictate system performance, we need to study it carefully.

27 **Reviewer 4:** Thank you for enumerating such a long list of strengths for our paper.

28 In regards to the background on dynamical systems, as you point out, we included materials both in the main part
29 as well as the full paper. Although the theoretical analysis is technical, the message itself is easily understood even
30 by non-experts. We believe that there is important value and insights both for theoretical minded people as well as
31 experimentalists, especially given our very thorough experimental study with numerous figures. We agree that NeurIPS
32 space constraints are tight and have forced us to push the presentation of some important results, as you point out, into
33 the appendix. Nevertheless, all of our results are at least described completely in the introduction and we hope that the
34 interested reader would benefit from our expansive supplement and directly jump in the section that they find more
35 interesting. Finally, we respectfully disagree that our paper is only tangentially related to NeurIPS. The Palaiopoulos et
36 al. paper was published at NeurIPS in 2017 with a spotlight distinction. Since then it has received 46 citations including
37 from at least 8 other NeurIPS papers. The ideas relating to period 3 inducing Li-Yorke chaos in dynamical systems have
38 since then found new applications related to the representational power of Deep Neural Networks: Chatziafratis et al.
39 Depth-Width Trade-offs for ReLU Networks via Sharkovsky's Theorem ICLR '20 (spotlight); Chatziafratis et al. Better
40 Depth-Width Trade-offs for Neural Networks through the lens of Dynamical Systems ICML '20.

41 We are very grateful for your detailed comments. We will incorporate them in an updated version of our paper.

42 **Reviewer 7:** Thank you for your support of our paper.

43 *Are there implications for MWU in practice?* In applications Exp3, the bandit version of MWU, is typically used.
44 Recent work on wireless network selection, which is a congestion game, has shown that Exp3 is unstable and has bad
45 performance necessitating new domain-specialized algs (Oh et al. Periodic Bandits and Wireless Network Selection
46 ICALP '19, Appavoo et al. Shrewd Selection Speeds Surfing: Use Smart EXP3! Int. Conf. on Distributed Computing
47 Systems '18). These issues are important and largely unsolved from a practical perspective.

48 *The studied games are relatively simple, even those games which the author claims the results can be extended to.* The
49 emergence of chaos is clearly a hardness/complexity type of result. Such results only increase in strength the simpler
50 the class of examples is. Even a single instance of a bad example suffices as, e.g., in Palaiopoulos et al. We instead, show
51 that *all* simple networks, no matter their costs, number of links, (except for the case of two links with equal costs) will
52 *all* exhibit chaos, *every single instance of them*. The simplicity, robustness of our games is a major strength of our paper.

53 Our regret formula is correct as $\min\{\sum_{n=1}^T \alpha N x_n, \sum_{n=1}^T \beta N (1 - x_n)\}$ captures the minimum aggregate cost over
54 time of the two paths. We need the knowledge of x_n to compute that.