



1 We thank the reviewers for their time and constructive feedback. Our work provides a "new contribution to HMMs
 2 and the SMC literature (R1)" in the form of "a novel and useful Generalised SMC framework for robust filtering in
 3 the presence of likelihood misspecification (R2)" that is "theoretically well grounded (R3)" and of "relevance to the
 4 NeurIPS community (R4)". Robust filtering in the M-open setting was an open problem that we have formally tackled.

5 **The proposed approach for selecting β (R1)** is heuristic, but practically effective as the predictive performance is
 6 closely related to the marginal likelihood which can also be used. Alternatives such as model fit can certainly be used as
 7 well. We believe using a training set with the same statistical properties as the intended data makes sense practically
 8 (and allows for online inference). We observed that using the same data did not alter performance. These points have
 9 now been clarified in the manuscript.

10 **Interest in $\pi^\beta(\phi)$ (R1):** We have now clarified that, in the Bissiri et al. (2016) framework β is a parameter of a specified
 11 loss function: a subjective (generalised) Bayesian choice characterising confidence in model correctness. π^β is then
 12 considered the "correct" generalisation of the posterior within the GBI framework.

13 **Theory & Corollary 1 (R1):** We agree that the arguments used here are standard and have now emphasised this in the
 14 manuscript. The intention is to demonstrate that existing arguments hold in our setting, not to claim novelty. We have
 15 added the proof of Corollary 1.

16 **β -divergence notation (R1):** We have now changed "small" to "positive" in 3.2 to convey the intended meaning.

17 **Additional evaluation (R1 & R2):** We thank the referees for their helpful suggestions, which have now been added
 18 and emphasised in the paper. Please see some examples of additional results. 1) Non-contaminated regime on Wiener
 19 velocity example (accidental omission) is shown on the left, where for moderate β the performance is near optimal. 2)
 20 Comparison to an oracle is one the right. 3) More complex likelihood and contamination in the form of an asymmetric,
 21 multiplicative noise model was presented in Appendix D due to space constraints.

22 **Broadening the support of the likelihood (R1 & R3):** changing the likelihood (e.g. by increasing the measurement
 23 variance) to decrease the influence of outliers suffers from drawbacks such as underestimating the influence of in-lying
 24 observations and difficulty in interpreting results. Using the beta-divergence is principled and decreases the influence of
 25 outliers while maintaining the influence on inliers (centre figure). We have included the influence function explanation.

26 **The relationship to parameter selection and model selection (R1 & R4)** have now been clearly signposted based
 27 on Kantas et al. (2015) and R4's suggestions, including elaborating on parameter estimation in our setting and the
 28 complications that arise in misspecified scenarios (cf. Brynjarsdóttir and O'Hagan, 2010). We consider challenging
 29 M-open settings: we do not assume access to a family of models which includes the true generative model. Consequently,
 30 model selection schemes cannot generally be used to correct for misspecification. We provide a formally justified
 31 procedure for SMC algorithms that is forgiving to likelihood misspecification and allows for inferring posterior belief
 32 distributions with desirable properties. This differs qualitatively from the listed literature in that the true model is not
 33 available among our candidate models. In Urteaga, et. al. information from many candidate models is fused according to
 34 their predictive performance, which is a pragmatic solution with good empirical performance on a univariate stochastic
 35 volatility model, when a good suite of candidates is available. We view this as complementary to our work.

36 **Novelty (R3):** In our view, the principal novelty lies in interpreting filtering within the framework of GBI (a previously
 37 unexplored direction) and hence arriving at a formally justified approach to robust filtering in the M-open setting.

38 **Examples of non-exponential family likelihoods (R1),** such as student-t likelihoods have now been added and
 39 discussed. Most stochastic volatility models could be addressed in closed form using properties of Gaussians.

40 **GP Hyperparameters (R1)** were selected by cross-validation. We have changed "known" to "fixed".

41 **Likelihood misspecification not model misspecification (R1 & R2):** We have changed the presentation and made the
 42 change in terminology advocated by R2; we indeed focus on likelihood misspecification although the GBI framework
 43 of Bissiri, et. al. can in principle be used more widely, which we have now also highlighted.

44 **Reproducibility (R4):** We have submitted the code as well as detailed descriptions of the experiments with the
 45 supplementary material. If anything has been omitted, please inform us in review and we would be happy to include it.