Supplementary Materials for
Cascaded Text Generation with Markov Transformers

Appendix A: Cascaded Decoding Examples

We show a decoding example in Table 3 ($K = 5$, $\Delta L = 1$, iters=5). We sort states by max-marginals in descending order and use - to denote invalid states (with $-\infty$ log max-marginals). In this simple sentence, using 1 iteration ($m = 0$, non-autoregressive model) repeats the word "woman" ($m = 0$, first row, $x_{4:4+m}$). Introducing higher order dependencies fixes this issue.

Table 3: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns "an amazing woman . eos". The source is "eine erstaunliche frau . eos" and the target is "an amazing woman . eos".

m	$x_{1:1+m}$	$x_{2:2+m}$	$x_{3:3+m}$	$x_{4:4+m}$	$x_{5:5+m}$	$x_{6:6+m}$	$x_{7:7+m}$	x_8
	an	amazing	woman	woman	eos	eos	eos	pad
	amazing	woman	amazing	.	.	pad	pad	-
0	incredible	an	an	amazing	woman	.	.	-
	this	remarkable	.	eos	amazing	woman	woman	-
	remarkable	incredible	women	an	women	women	women	-
	an amazing	amazing woman	woman .	. eos	eos pad	pad pad	pad pad	
	an incredible	incredible woman	amazing woman	woman .	. eos	eos pad	eos pad	
1	this amazing	remarkable woman	women .	amazing woman	woman .	. eos	-	
	an remarkable	woman amazing	woman woman	. .	women .	woman eos	-	
	amazing woman	amazing women	an amazing	. woman	. .	-	-	
	an amazing woman	amazing woman .	woman .eos	. eos pad	eos pad pad	pad pad pad		
	an incredible woman	incredible woman .	women . eos	woman . eos	. eos pad	eos pad pad		
2	this amazing woman	remarkable woman .	woman woman .	. . eos	woman . eos	. eos pad		
	an remarkable woman	amazing women .	woman . .	. woman .	. . eos	-		
	an amazing women	amazing woman woman	woman . woman	woman . .	-	-		
	an amazing woman .	amazing woman . eos	woman . eos pad	. eos pad pad	eos pad pad pad			
	an incredible woman .	incredible woman . eos	women . eos pad	woman . eos pad	. eos pad pad			
3	this amazing woman .	remarkable woman . eos	woman woman . eos	. . eos pad	woman . eos pad			
	an remarkable woman .	amazing women . eos	woman . . eos	. woman . eos	. . eos pad			
	an amazing women .	amazing woman woman .	woman . woman .	woman . . eos	-			

Table 4: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns "what has happened ? eos". The source is "was ist passiert ? eos" and the target is "what happened ? eos".

m	$x_{1:1+m}$	$x_{2:2+m}$	$x_{3:3+m}$	$x_{4:4+m}$	$x_{5:5+m}$	$x_{6:6+m}$	$x_{7:7+m}$	x_8
	what	happened	happened	?	eos	eos	eos	pad
	so	has	?	eos	?	pad	pad	-
0	now	did	what	happened	happened	?	?	-
	and	what	happen	happen	happen	happened	.	-
	well	’s	eos	happens	happens	.	happened	-
	what has	has happened	happened ?	? eos	eos pad	pad pad	pad pad	
	so what	what happened	what happened	happened ?	? eos	eos pad	eos pad	
1	and what	’s happened	happen ?	happens ?	happens ?	? eos	-	
	what ’s	did what	what ?	happen ?	happen ?	. eos	-	
	now what	did happened	what happens	? ?	happened ?	happened eos	-	
	what has happened	has happened ?	happened ? eos	? eos pad	eos pad pad	pad pad pad		
	so what happened	what happened ?	happened ? ?	? ? eos	? eos pad	eos pad pad		
2	what ’s happened	’s happened ?	happen ? ?	happen ? eos	happened ? eos	happened eos pad		
	and what happened	did what ?	happen ? eos	happens ? eos	happen ? eos	. eos pad		
	now what happened	did what happened	what happened ?	happened ? eos	happens ? eos	? eos pad		
	what has happened ?	has happened ? eos	happened ? eos pad	? eos pad pad	eos pad pad pad			
	so what happened ?	what happened ? eos	happened ? ? eos	? ? eos pad	? eos pad pad			
3	and what happened ?	’s happened ? eos	what happened ? eos	happened ? eos pad	happens ? eos pad			
	what ’s happened ?	has happened ? ?	happen ? eos pad	happens ? eos pad	happen ? eos pad			
	now what happened ?	what happened ? ?	happen ? ? eos	happen ? eos pad	happened ? eos pad			

In Tables 4, 5, 6, 7, 8 we show more examples from IWSLT14 De-En val.

Table 5: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns “you ’re happy . eos”. The source is “du bist glücklich . eos” and the target is “you ’re happy . eos”.

m	$x_{1:1+m}$	$x_{2:2+m}$	$x_{3:3+m}$	$x_{4:4+m}$	$x_{5:5+m}$	$x_{6:6+m}$	$x_{7:7+m}$	x_8
	you	’re	happy	.	eos	eos	eos	pad
	happy	are	lucky	eos	.	pad	pad	-
0	your	you	gla@@	happy	happy	.	.	-
	and	’s	good	lucky	?	happy	happy	-
	i	be	fortun@@	ful	you	?	?	-
	you ’re	’re happy	happy .	. eos	eos pad	pad pad	pad pad	
	you are	are happy	lucky .	. .	. eos	eos pad	eos pad	
1	you be	are lucky	good .	happy .	happy .	. eos	-	
	you ’s	be happy	happy happy	ful .	? eos	? eos	-	
	and you	’re lucky	happy ful	lucky .	you .	happy eos	-	
	you ’re happy	’re happy .	happy . eos	. eos pad	eos pad pad	pad pad pad		
	you are happy	are happy .	lucky . eos	. . eos	. eos pad	eos pad pad		
2	you be happy	be happy .	happy . .	happy . eos	you . eos	happy eos pad		
	you ’re lucky	’re lucky .	happy happy .	ful . eos	? eos pad	? eos pad		
	you are lucky	are lucky .	happy ful .	lucky . eos	happy . eos	. eos pad		
	you ’re happy .	’re happy . eos	happy . eos pad	. eos pad pad	eos pad pad pad			
	you are happy .	are happy . eos	lucky . eos pad	. . eos pad	. eos pad pad			
3	you be happy .	be happy . eos	happy . . eos	lucky . eos pad	happy . eos pad			
	you ’re lucky .	’re lucky . eos	happy ful . eos	ful . eos pad	? eos pad pad			
	you are lucky .	are lucky . eos	happy happy . eos	happy . eos pad	you . eos pad			

Table 6: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns “let ’s move . eos”. The source is “bewe@@ g dich . eos” and the target is “move it . eos”.

m	$x_{1:1+m}$	$x_{2:2+m}$	$x_{3:3+m}$	$x_{4:4+m}$	$x_{5:5+m}$	$x_{6:6+m}$	$x_{7:7+m}$	x_8
	move	move	.	eos	eos	eos	eos	pad
	let	.	eos	.	.	pad	pad	-
0	so	moving	move	?	?	.	.	-
	just	’s	forward	forward	here	?	?	-
	now	let	moving	it	forward	here	here	-
	let ’s	’s move	move .	. eos	eos pad	pad pad	pad pad	
	just move	’s moving	moving .	it .	. eos	eos pad	eos pad	
1	so move	move forward	move it	forward .	here .	. eos	-	
	move .	. forward	move forward	? eos	? eos	? eos	-	
	move ’s	. moving	move ?	. .	forward .	-	-	
	let ’s move	’s move .	move . eos	. eos pad	eos pad pad	pad pad pad		
	let ’s moving	’s move it	move it .	it . eos	. eos pad	eos pad pad		
2	move ’s move	’s move forward	move forward .	forward . eos	? eos pad	? eos pad		
	move . moving	’s moving .	moving . eos	? eos pad	here . eos	. eos pad		
	move ’s moving	’s move ?	move ? eos	. . eos	-	-		
	let ’s move .	’s move . eos	move . eos pad	. eos pad pad	eos pad pad pad			
	let ’s move it	’s move it .	move it . eos	it . eos pad	. eos pad pad			
3	let ’s moving .	’s moving . eos	moving . eos pad	forward . eos pad	here . eos pad			
	let ’s move forward	’s move forward .	move forward . eos	? eos pad pad	? eos pad pad			
	let ’s move ?	’s move ? eos	move ? eos pad	. . eos pad	-			

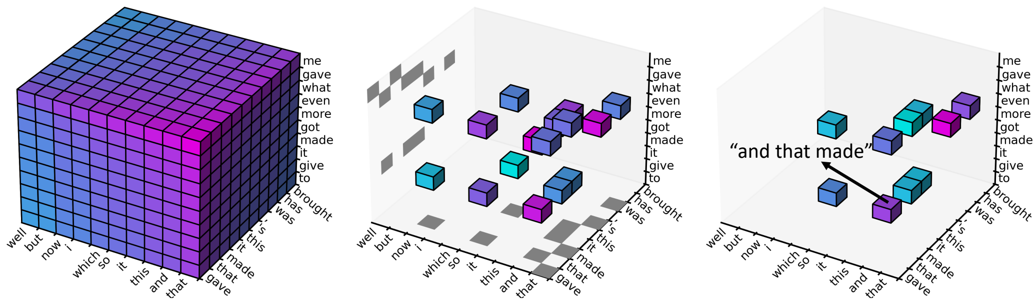
Table 7: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns "very , very hard . eos". The source is "sehr sehr schwer . eos" and the target is "very very hard . eos".

m	$x_{1:1+m}$	$x_{2:2+m}$	$x_{3:3+m}$	$x_{4:4+m}$	$x_{5:5+m}$	$x_{6:6+m}$	$x_{7:7+m}$	x_8
	very	difficult	difficult	.	eos	eos	eos	pad
	it	hard	hard	eos	.	pad	pad	-
0	really	very	.	difficult	difficult	.	.	-
	extremely	tough	very	hard	hard	difficult	difficult	-
	that	,	tough	very	very	hard	hard	-
	very ,	, very	very difficult	difficult .	. eos	eos pad	pad pad	
	very very	very hard	very hard	hard .	eos pad	pad pad	eos pad	
1	really ,	very difficult	hard .	. eos	difficult .	. eos	-	
	it very	, hard	difficult .	hard eos	hard .	difficult eos	-	
	extremely ,	, difficult	tough .	difficult eos	. .	hard eos	-	
	very , very	, very hard	very hard .	hard . eos	. eos pad	eos pad pad		
	very very difficult	, very difficult	very difficult .	difficult . eos	eos pad pad	pad pad pad		
2	very very hard	very difficult .	difficult . eos	. eos pad	. . eos	. eos pad		
	really , very	very hard .	hard . eos	hard eos pad	hard . eos	hard eos pad		
	it very difficult	, hard .	very hard eos	difficult eos pad	difficult . eos	difficult eos pad		
	very , very hard	, very hard .	very hard . eos	hard . eos pad	. eos pad pad			
	very , very difficult	, very difficult .	very difficult . eos	difficult . eos pad	eos pad pad pad			
3	very very difficult .	very difficult . eos	difficult . eos pad	. eos pad pad	difficult . eos pad			
	very very hard .	very hard . eos	hard . eos pad	hard eos pad pad	hard . eos pad			
	really , very hard	, very hard eos	very hard eos pad	difficult eos pad pad	. . eos pad			

Table 8: Cascaded Decoding Example. When $m = 4$, Viterbi in $\mathcal{X}_4$ returns "the opposite thing happened . eos". The source is "das gegenteil passierte . eos" and the target is "the opposite happened . eos".

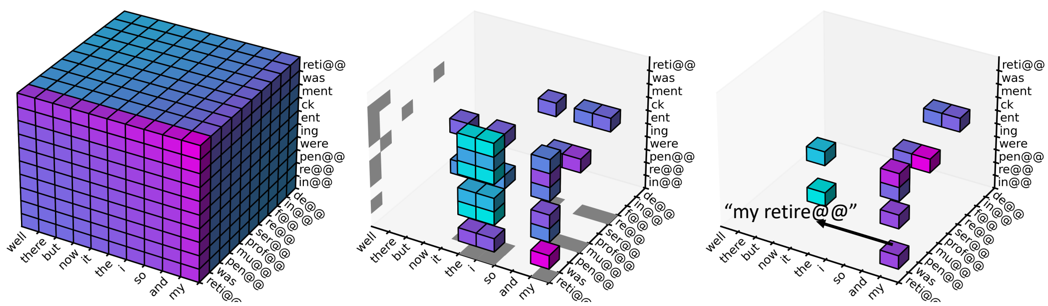
m	$x_{1:1+m}$	$x_{2:2+m}$	$x_{3:3+m}$	$x_{4:4+m}$	$x_{5:5+m}$	$x_{6:6+m}$	$x_{7:7+m}$	x_8
	the	opposite	opposite	happened	eos	eos	eos	pad
	and	contr@@	thing	was	.	pad	pad	-
0	so	other	ary	thing	happened	.	.	-
	but	the	happened	did	happening	happened	happened	-
	well	conver@@	was	opposite	happen	happen	happen	-
	the opposite	opposite thing	thing happened	happened .	. eos	eos pad	pad pad	
	the contr@@	contr@@ ary	ary happened	was happening	happening .	. eos	eos pad	
1	and the	the opposite	opposite happened	thing happened	happened .	pad pad	-	
	the other	other thing	thing was	did .	eos pad	. happened eos	-	
	so the	opposite opposite	was happened	was happened	. .	-	-	
	the opposite thing	opposite thing happened	thing happened .	happened . eos	. eos pad	eos pad pad		
	the contr@@ ary	contr@@ ary happened	ary happened .	was happening .	happening . eos	. eos pad		
2	and the opposite	the opposite happened	opposite happened .	was happened .	happened . eos	happened eos pad		
	the other thing	other thing happened	thing was happening	happened . .	. . eos	pad pad pad		
	so the opposite	opposite thing was	thing was happened	thing happened .	-	-		
	the opposite thing happened	opposite thing happened .	thing happened . eos	happened . eos pad	. eos pad pad			
	the contr@@ ary happened	contr@@ ary happened .	ary happened . eos	was happening . eos	happening . eos pad			
3	and the opposite happened	the opposite happened .	opposite happened . eos	was happened . eos	happened . eos pad			
	the other thing happened	other thing happened .	thing was happening .	happened . . eos	. . eos pad			
	the opposite thing was	opposite thing was happening	thing was happened .	-	-			

Appendix B: More Visualizations



(a) $m = 0$ (b) $m = 1$ (c) $m = 2$

Figure 4: Illustration of cascaded decoding ($K = 10$, iters=4) for $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$.



(a) $m = 0$ (b) $m = 1$ (c) $m = 2$

Figure 5: Illustration of cascaded decoding ($K = 10$, iters=4) for $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$.

We include more visualizations of $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{X}_3$ in Figure 4 and Figure 5. These examples are taken from IWSLT14 De-En val.

Appendix C: Variable Length Generation Potentials

To handle length, we introduce an additional padding symbol pad to $\mathcal{V}$, and change the log potentials to enforce the considered candidates are of length $L - \Delta L$ to $L + \Delta L$. Note that we can only enforce that for $m \geq 1$, and for $m = 0$ we manually add pad to the pruned vocabulary.

We start cascaded search using a sequence of length $L + \Delta L + 1$. The main ideas are: 1) We make eos and pad to always transition to pad such that sequences of different lengths can be compared; 2) We disallow eos to appear too early or too late to satisfy the length constraint; 3) We force the last token to be pad such that we don't end up with sentences without eos endings. Putting these ideas together, the modified log potentials we use are:

$$
f'^{(m)}_l(x_{l:l+m}) = \begin{cases} 0, \text{ if } x_{l+m-1} = \texttt{eos} \wedge x_{l+m} = \texttt{pad} \\ -\infty, \text{ if } x_{l+m-1} = \texttt{eos} \wedge x_{l+m} \neq \texttt{pad} \text{ (}\texttt{eos} \rightarrow \texttt{pad}\text{)} \\ 0, \text{ if } x_{l+m-1} = \texttt{pad} \wedge x_{l+m} = \texttt{pad} \\ -\infty, \text{ if } x_{l+m-1} = \texttt{pad} \wedge x_{l+m} \neq \texttt{pad} \text{ (}\texttt{pad} \rightarrow \texttt{pad}\text{)} \\ -\infty, \text{ if } x_{l+m-1} \neq \texttt{pad} \wedge x_{l+m-1} \neq \texttt{eos} \wedge x_{l+m} = \texttt{pad} \text{ (nothing else} \rightarrow \texttt{pad}\text{)} \\ -\infty, \text{ if } l+m < L - \Delta L \wedge x_{l+m} = \texttt{eos} \text{ (}\texttt{eos}\text{ cannot appear too early)} \\ 0, \text{ if } l+m = L + \Delta L + 1 \text{ and } x_{l+m} = \texttt{pad} \\ -\infty, \text{ if } l+m = L + \Delta L + 1 \text{ and } x_{l+m} \neq \texttt{pad} \text{ (the last token must be }\texttt{pad}\text{)} \\ f^{(m)}_l(x_{l:l+m}), \text{ o.t.} \end{cases} .
$$

Note that we only considered a single sentence above, but batching is straightforward to implement and we refer interested readers to our code[5] for batch implementations.

Appendix D: Full Results

In the main experiment table we showed latency/speedup results for WMT14 En-De. In Table 9, Table 10, Table 11 and Table 12 we show the latency/speedup results for other datasets. Same as in the main experiment table, we use the validation set to choose the configuration with the best BLEU score under speedup $> \times 1$, $> \times 2$, etc.

Table 9: Results on WMT14 De-En.

Model	Settings	Latency (Speedup)	BLEU
Transformer	(beam 5)	294.64ms (×1.00)	31.49
With Distillation			
Cascaded Generation *with Speedup*			
$> \times 7$	(K=16, iters=2)	43.41ms (×6.79)	30.69
$> \times 6$	(K=32, iters=2)	52.06ms (×5.66)	30.72
$> \times 5$	(K=16, iters=3)	62.06ms (×4.75)	30.96
$> \times 4/3$	(K=32, iters=3)	79.01ms (×3.73)	31.08
$> \times 2/1$	(K=32, iters=5)	129.67ms (×2.27)	31.15
Without Distillation			
Cascaded Generation *with Speedup*			
$> \times 6/5$	(K=32, iters=2)	53.83ms (×5.47)	27.56
$> \times 4$	(K=32, iters=3)	81.10ms (×3.63)	28.64
$> \times 3$	(K=32, iters=4)	106.97ms (×2.75)	28.73
$> \times 2$	(K=64, iters=4)	154.15ms (×1.91)	29.43
$> \times 1$	(K=128, iters=4)	269.59ms (×1.09)	29.66

[5] https://github.com/harvardnlp/cascaded-generation

Table 10: Results on WMT16 En-Ro.

Model	Settings	Latency (Speedup)	BLEU
Transformer	(beam 5)	343.28ms ($\times 1.00$)	33.89
With Distillation			
Cascaded Generation *with Speedup*			
$> \times 7$	(K=16, iters=2)	49.38ms ($\times 6.95$)	32.70
$> \times 6$	(K=32, iters=2)	54.56ms ($\times 6.29$)	32.73
$> \times 5$	(K=16, iters=3)	66.33ms ($\times 5.18$)	32.89
$> \times 4$	(K=32, iters=3)	77.39ms ($\times 4.44$)	33.16
$> \times 3$	(K=64, iters=3)	108.57ms ($\times 3.16$)	33.23
$> \times 2$	(K=64, iters=4)	142.23ms ($\times 2.41$)	33.30
$> \times 1$	(K=64, iters=5)	179.07ms ($\times 1.92$)	33.23
Without Distillation			
Cascaded Generation *with Speedup*			
$> \times 7$	(K=16, iters=2)	45.18ms ($\times 7.60$)	32.11
$> \times 6$	(K=32, iters=2)	51.38ms ($\times 6.68$)	32.62
$> \times 5$	(K=16, iters=3)	60.34ms ($\times 5.69$)	32.67
$> \times 4$	(K=32, iters=3)	73.99ms ($\times 4.64$)	33.12
$> \times 3$	(K=64, iters=3)	105.46ms ($\times 3.26$)	33.48
$> \times 2$	(K=64, iters=4)	145.18ms ($\times 2.36$)	33.64
$> \times 1$	(K=128, iters=5)	325.42ms ($\times 1.05$)	33.52

Table 11: Results on WMT16 Ro-En.

Model	Settings	Latency (Speedup)	BLEU
Transformer	(beam 5)	318.57ms ($\times 1.00$)	33.82
With Distillation			
Cascaded Generation *with Speedup*			
$> \times 6/5$	(K=16, iters=2)	46.84ms ($\times 6.80$)	32.66
$> \times 4$	(K=16, iters=3)	62.57ms ($\times 5.09$)	33.00
$> \times 3$	(K=16, iters=5)	99.25ms ($\times 3.21$)	33.04
$> \times 2$	(K=64, iters=3)	103.85ms ($\times 3.07$)	33.17
$> \times 1$	(K=64, iters=5)	181.18ms ($\times 1.76$)	33.28
Without Distillation			
Cascaded Generation *with Speedup*			
$> \times 6$	(K=16, iters=2)	47.58ms ($\times 6.70$)	32.53
$> \times 5$	(K=32, iters=2)	54.05ms ($\times 5.89$)	32.44
$> \times 4$	(K=16, iters=3)	60.94ms ($\times 5.23$)	33.00
$> \times 3$	(K=32, iters=4)	100.29ms ($\times 3.18$)	33.10
$> \times 2$	(K=64, iters=3)	105.21ms ($\times 3.03$)	33.22
$> \times 1$	(K=128, iters=4)	282.76ms ($\times 1.13$)	33.29

Table 12: Results on IWSLT14 De-En.

Model	Settings	Latency (Speedup)	BLEU
Transformer	(beam 5)	229.76ms ($\times 1.00$)	34.44
With Distillation			
Cascaded Generation *with Speedup*			
$> \times 6/5$	(K=16, iters=2)	39.38ms ($\times 5.83$)	33.90
$> \times 4$	(K=32, iters=3)	60.27ms ($\times 3.81$)	34.33
$> \times 3$	(K=32, iters=4)	78.27ms ($\times 2.94$)	34.43
$> \times 2/1$	(K=64, iters=5)	117.90ms ($\times 1.95$)	34.49
Without Distillation			
Cascaded Generation *with Speedup*			
$> \times 5$	(K=64, iters=2)	48.59ms ($\times 4.73$)	33.25
$> \times 4$	(K=32, iters=3)	60.09ms ($\times 3.82$)	33.74
$> \times 3$	(K=64, iters=3)	75.64ms ($\times 3.04$)	33.96
$> \times 2$	(K=64, iters=5)	121.95ms ($\times 1.88$)	34.08
$> \times 1$	(K=128, iters=5)	189.10ms ($\times 1.22$)	34.15

Appendix E: Optimization Settings

Table 13: Optimization settings. We use the same settings for knowledge distillation experiments.

Dataset	dropout	fp16	GPUs	batch	accum	warmup steps	max steps	max lr	weight decay
WMT14 En-De/De-En	0.1	Y	3	4096	3	4k	240k	7e-4	0
WMT16 En-Ro/Ro-En	0.3	Y	3	5461	1	10k	240k	7e-4	1e-2
IWSLT14 De-En	0.3	N	1	4096	1	4k	120k	5e-4	1e-4

Our approach is implemented in PyTorch [35], and we use 16GB Nvidia V100 GPUs for training. We used Adam optimizer [20], with betas 0.9 and 0.98. We use inverse square root learning rate decay after warmup steps [34]. We train with label smoothing strength 0.1 [32]. For model selection, we used BLEU score on validation set. For Markov transformers, we use cascaded decoding with $K = 16$ and $\Delta L = 3$ to compute validation BLEU score. Other hyperparameters can be found at Table 13.