

1 We thank the reviewers for their interest, and hope this document clarifies our unique contributions.  
2 In the final version, we will include additional exposition addressing each of the following points.

3 Reviewer #2 questions whether it is appropriate to describe our method as a new mechanism, since  
4 it can be viewed as a particular case of the exponential mechanism. In fact many widely used DP  
5 mechanisms can be expressed as an instance of the exponential mechanism, such as the Laplace  
6 mechanism, geometric mechanism, staircase mechanism,  $K$ -norm mechanism, posterior sampling  
7 mechanism, etc. As we show, the KNG approach is applicable in a wide variety of situations, and  
8 offers improved utility over a classic implementation of the exponential mechanism. For these reasons,  
9 we think it is justified to refer to KNG as its own mechanism.

10 Reviewer #3 asks for intuition behind the use of the gradient in KNG. The proof of the CLT for the  
11 exponential mechanism in Awan et al. 2019 (ICML), as well as the proof of Theorem 3.2, both rely  
12 on a Taylor expansion of the objective function. In both cases, it is assumed that the hessian converges  
13 at a  $O(n)$  rate to a positive definite matrix. However, using the original objective function requires  
14 two derivatives before the Hessian appears in the Taylor expansion, whereas the use of the gradient  
15 only requires one derivative. The consequence of this is that the traditional exponential mechanism  
16 results in a quadratic numerator, whereas KNG has a (normed) linear numerator. Asymptotically, this  
17 gives  $O(1/\sqrt{n})$  Gaussian noise for the exponential mechanism and  $O(1/n)$   $K$ -norm noise for KNG.

18 Geometrically, it seems that the use of an objective function which behaves linearly (in absolute  
19 value) near the optimum, rather than quadratic, results in better asymptotic utility. By using the  
20 normed-gradient, we construct an objective function with this property.

21 Reviewer #4 asks for clarification on how the performance of KNG differs from the exponential  
22 mechanism. While it may not have been clear in the exposition of our submission, in fact the  
23 assumptions in THM 3.2 are nearly identical to the assumptions required in the CLT of Awan et al.  
24 2019 (ICML). So, for any problem in which these assumptions are satisfied, KNG always results in  
25  $O(1/n)$  noise, whereas exponential mechanism results in  $O(1/\sqrt{n})$  noise.

26 Reviewer #4 also asks for interesting problems where KNG outperforms the exponential mechanism.  
27 Among the examples in the manuscript, mean estimation and linear regression both satisfy all of the  
28 assumptions to justify that KNG results in  $O(1/n)$  noise, whereas exponential mechanism results in  
29  $O(1/\sqrt{n})$  noise. While the problems of median estimation and quantile regression do not satisfy the  
30 assumptions of THM 3.2 (though they are still private), we demonstrated empirically via simulations  
31 that KNG still results in  $O(1/n)$  whereas exponential mechanism results in  $O(1/\sqrt{n})$  noise.

32 To emphasize the improvement that KNG offers over the exponential mechanism, we point out  
33 that adding  $O(1/n)$  versus  $O(1/\sqrt{n})$  noise has a substantial impact on the sample complexity.  
34 Asymptotically, KNG requires exactly the same sample size as the non-private estimator, whereas  
35 exponential mechanism requires a constant  $>1$  multiple of the non-private sample size.

36 We also note that THM 3.2, as well as the examples of median estimation and quantile regression,  
37 indicate that KNG outperforms the exponential mechanism for a wide variety of interesting problems.  
38 Many log-likelihoods fit this framework, as well as many other empirical risk functions.

39 Reviewer #4 also asks about sampling algorithms for the KNG mechanisms. KNG is similar to the  
40 exponential mechanism in that sampling these distributions is generally non-trivial. We show that for  
41 mean and quantile estimation, KNG results in distributions that are efficiently sampled. However, for  
42 linear and quantile regression, we used a one-at-a-time MCMC procedure (also used for exponential  
43 mechanism). Just like sampling from an posterior distribution, developing a convenient sampling  
44 scheme is case-by-case, but often a simple MCMC procedure works well in practice.

45 Finally, Reviewer #4 questions whether the results in this manuscript are significant enough for  
46 publication in NeurIPS. While we acknowledge that the proof of THM 3.2 is not technically com-  
47 plex, we argue that KNG offers both an important theoretical and practical contribution to the DP  
48 literature. While it has been shown before that asymptotically efficient mechanisms exist [Smith  
49 2011], constructing practical and efficient mechanisms for a particular problem is non-trivial. KNG  
50 offers a principled approach to developing efficient mechanisms. For mean and quantile estimation,  
51 KNG offers a method of constructing both the Laplace and PrivateQuantile mechanisms. However,  
52 besides unifying these prior mechanisms, KNG can also be used to build mechanisms for problems  
53 not previously solved. In fact, using KNG we develop the first DP mechanism for quantile regression  
54 that we are aware of, and demonstrate empirically that the mechanism is asymptotically efficient.