- We appreciate the reviewers' support for the novelty, remarkable performance, and the impact of our work. We thank the
- reviewers for their valuable feedback and thoughtful reviews. Below we address the concerns raised by the reviewers.
- **Reviewer 1:** It would be great to compare inference time and memory consumption against other defense methods.
- **Reply:** Our defense method has negligible overhead in inference time and memory compared with inference by the
- standard ResNet without defense, and this is also the case for the PGD adversarial training (arXiv:1706.06083) and
- TRADES (arXiv:1901.08573). For CIFAR10, the inference time for ResNet20 and En₁ResNet20, averaged over 100
- runs, with batch size 1K on a Titan Xp are 1.6941s and 1.6943s, resp. The corresponding peak memory is 4807MB for
- both ResNet20 and En₁ResNet20. We will report the inference time and memory in the revised manuscript. 8
- **Reviewer 1:** In Fig. 4, it is unclear why neural networks made wrong predictions. It could be due to the difficulty of 9 the input example or adversarial attack. To make it more clear, authors need to add prediction results for the clean input. 10
- **Reply:** We will add the classification results for the clean input in the revised manuscript. 11
- **Reviewer 1:** I do not see experiments that can support claim that this method is a complement to existing defenses. 12
- Reply: Directly replacing the cross-entropy loss with the TRADES loss can further improve the robust accuracy, under 13
- the IFGSM²⁰ attack, of the WideResNet (arXiv:1901.08573) by $\sim 0.9\%$. We will report more results in the revision. 14
- **Reviewer 1:** The method is inherently limited to ResNet based network architectures. 15
- **Reply:** The EnResNet is motivated from the Euler-Maruyama discretization of the Itô process below Eq. (10). Other 16 numerical discretization may motivate ensemble of new network architectures like neural ODE as our future work. 17
- **Reviewer 3:** The authors are probably not aware of some similar papers like [1,2,3]. 18
- **Reply:** We will discuss these papers in the related work Sec. in the revision. 19

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- **Reviewer 3:** The proposed PDE formalism and the resulted method does not depend on the training objective function. 20
- Therefore, Theorem 1 should provide some degree of adversarial robustness even in the absence of adversarial training. 21
- Still, empirical results regarding the performance of the proposed method with natural training are missing.
- **Reply:** For natural training, En₁ResNet20 and En₂ResNet20 (with injected Gaussian noise of standard deviation 0.1)
- has accuracy 27.93% and 28.75%, resp., under the FGSM attack with $\epsilon = 8/255$, in contrast to ResNet20 with robust 24
- accuracy 10.45% under the same attack. Moreover, as shown in Sec. 3.3, the naturally trained En_n ResNet20 has slightly 25
- better robust accuracy than ResNet20 under the IFGSM 20 attack. Adversarial training enables En_nResNets to have remarkably better robust accuracy than ResNets under the IFGSM 20 attack. We will make this clear in the revision. 26 27
- Reviewer 3: I wonder if the ResNets are in a single ensemble sharing weights. If not, the implementation is not 28 consistent with the proposed PDE formalism. If yes, I would like to make sure that the share-weights ensemble (SWE) of ResNets truly outperforms standard ResNet on natural examples.
- Reply: In our paper, EnResNets do not share weights. Direct ResNet ensemble counterpart of the PDE formalism 31
- needs to share weights. Table 1 below shows that, SWE also improves both natural and robust accuracies which verifies 32 the efficacy of our PDE formalism. Moreover, to further improve the ensemble model's performance, we generalize
- SWE to non-share weights ensemble (NSWE) with the consideration of increasing the model capacity. As shown in 34
- Table 1 in our paper (En₂ResNet20 v.s. ResNet44), NSWE remarkably outperforms the vanilla ResNets with a similar 35 capacity. We will point out this and include the results of SWE in the revision.

Table 1: Accuracy of the robustly trained $n \times \text{En}_1 \text{ResNet20}$ which denotes the ensemble of n share-weights $\text{En}_1 \text{ResNet20}$.

	ResNet20	$1 \times \text{En}_1\text{ResNet20}$	$2 \times \text{En}_1\text{ResNet20}$	$5 \times \text{En}_1 \text{ResNet20}$
$egin{aligned} \mathcal{A}_{\mathrm{nat}} \ \mathcal{A}_{\mathrm{rob}} \ (ext{IFGSM}^{20}) \end{aligned}$	75.11%	77.21%	77.88%	77.99%
	46.03%	49.06%	49.17%	49.20%

- **Reviewer 3:** Do the authors run the experiments for multiple times observe consistent gain over the baseline? 37
- Reply: The reported accuracies are averaged over five runs, and the standard deviation is less than 0.5% among these 38 runs. The accuracy of EnResNets is consistently better than the baseline over different runs. 39
- **Reviewers 3 & 4:** Influence of ϵ for adversarial perturbation, and the standard deviation of the Gaussian noise. 40
- **Reply:** We take the most-used ϵ , and different ϵ /noise will be discussed in the revision. Table 2 lists \mathcal{A}_{rob} (IFGSM²⁰) 41
- of the robustly trained En₂ResNet20 with different noise. 0.1 gives a good trade-off between accuracy and variance.

Table 2: Robust accuracy of the PGD adversarially trained En₂ResNet20 with different Gaussian noise injection. (five runs)

<u></u>	<u>-</u>			
Standard derivation (Gaussian noise)	0.05	0.1	0.4	0.8
$\mathcal{A}_{\mathrm{rob}}$ (IFGSM 20)	$50.05\% \pm 0.27\%$	$50.06\% \pm 0.35\%$	$50.51\% \pm 0.90\%$	$43.51\% \pm 3.78\%$

- Reviewers 3 & 4: 1). Specify the details of ResNets. 2). Reorganize Sec. 3.3. 3). Provide derivation of Eq. (10). 4). 43
- Line 126, better symbols for noise injected ResNet might be Resnet* etc. 5). Add the meanings of the symbols in Th 1. 44
- **Reply:** We will modify our manuscript to include all these valuable suggestions. 45
- **Reviewer 4:** The highest vanilla resnet accuracy in the paper is close to 85%, however, there are many implementations 46
- of resnets which achieve $\sim 95\%$ on CIFAR10. Why aren't the baseline numbers reported in the paper high enough? 47
- **Reply:** The reported accuracies are that of the robustly trained models by solving the EARM (Eq.(1)). The proposed 48
- ResNets ensemble can also improve the natural accuracy of the naturally trained models, e.g., $A_{\rm nat}$ of naturally trained
- En₂ResNet20 is 92.60% which is remarkably better than that of ResNet20 reported in He et al., arXiv:1512.03385.