

1 **Reviewer #1. 1) Comparisons** with “From PAC to Instance-Optimal Sample Complexity in the Plackett-Luce Model”  
2 by Saha and Gopalan (denoted as [SG]). We were not aware of this paper at the time of submission. Thanks for bringing  
3 it to our attention. After reading this paper, we found that our work is different from [SG] in the following aspects.  
4 (i) Our paper studies full ranking while [SG] focuses on maxing, which is a different problem. (ii) The algorithms in  
5 [SG] only work for the PL model, while our algorithm works for a larger class that strictly contains the pairwise PL  
6 model as a special case. The algorithms in [SG] depend on the special properties of the PL model, while ours does not.  
7 (iii) In [SG], the algorithm checks whether an item is (PAC-)less preferred than another one by comparing the empirical  
8  $p_{i,j}$ -value and  $(1/2 - \epsilon_s)$ . In contrast, our algorithm needs to check whether an item can be inserted to a sorted list with  
9 enough confidence, which can hardly be done by simply comparing empirical means. For this purpose, we constructed  
10 a subroutine ATC to replace the comparisons in Binary Search in [14] and redesigned the random walk procedure to get  
11 an attempting version of Binary Search. (iv) We also proved several lower bounds that are significant for understanding  
12 the full ranking problem. Further, to the best of our knowledge, we are the first to introduce the  $\Omega(\Delta_i^{-2} \log \log \Delta_i^{-1})$   
13 lower bound to the field of active ranking. **2) PAC vs. exact ranking.** (i) In some applications, we may want to find  
14 the exact order, especially in “winner-takes-all” situations. For example, when predicting the winner of an election,  
15 we prefer to get the exact result but not the PAC one, as only a few votes can completely change the result. (ii) Our  
16 algorithm IIR can utilize unequal noise levels, while previous PAC algorithms may not. For example, Refs [10,11,12]  
17 only depend on  $n\epsilon^{-2}$ . (iii) Studying exact ranking is theoretical significant and helps obtain insights on instance-wise  
18 upper and lower bounds. Specifically, the PAC bounds that depend on  $\epsilon^{-1}$  will become vacuous as  $\epsilon$  shrinks to zero,  
19 i.e., the PAC bounds do not reduce to meaningful instance-wise bounds when one pushes  $\epsilon$  to 0.

20 **Reviewer #2. 1) Independence.** (i) We believe assuming independence is realistic. For example, for ranking movies,  
21 it is reasonable to define  $p_{i,j}$  as the fraction of users that prefer movie  $i$  to  $j$ , and a comparison between  $i$  and  $j$  refers to  
22 sampling a user’s preference. The comparisons can be assumed to be independent if the users are randomly sampled.  
23 (ii) Independence across time, items, and sets is a common assumption in this field. (iii) If the comparisons were not  
24 independent, we might not be able to recover the true preference of the users. For example, if all annotators compare  
25 the items based on the comparison results of some user, then the ranking got from these comparisons may not correctly  
26 represent the preference of all users. **2) Notation of  $p_{i,j}$ .** For a comparison over set  $S$ , we use  $p_{i,S}$  to denote the  
27 probability that  $i$  wins this comparison. When  $|S| = 2$ , we write  $p_{i,j} = p_{i,\{i,j\}}$  to simplify notation, and  $\Delta_{i,j}$  is  
28 defined as  $|p_{i,j} - 1/2|$ . **3) Computational complexity.** (i) It is hard to quantify the actual computational complexity  
29 of a comparison as it can vary with applications. For example, a comparison may refer to asking an annotator to  
30 compare some items, which can take minutes or longer. Also, a comparison may refer to comparing random numerical  
31 values in computer simulations, which may only take milliseconds. Further, due to similar reasons, the difference  
32 between the computational complexities of pairwise comparisons and listwise comparisons is also hard to quantify.  
33 (ii) It is reasonable to assume that an  $m$ -wise comparison takes  $\Omega(m)$  time to pass the items to compare. Thus, by  
34 Proposition 7, we get that ranking  $n$  items with  $m$ -wise comparisons takes  $\Omega(mn \log_m n)$  time. From this perspective,  
35 pairwise ranking uses less time. **4) Worst case scenario.** (i) The MNL model is a widely used model. Thus, the lower  
36 bound for this model is significant in its own right. (ii) We analyzed the listwise ranking bounds for several cases not  
37 presented in this paper, and found that whether listwise ranking uses less samples depends on how we define the gaps of  
38 listwise comparisons. We believe the general listwise ranking problem deserves an independent paper. **5) Comparisons.**  
39 Comparisons may refer to querying users about their preferences, asking annotators to compare items, or retrieving  
40 data from a data center. We view comparisons as black-box procedures in this paper. **6) Assumptions.** For A1, please  
41 refer to “1) Independence” above. If A2 is not true, then the unique true ranking does not exist, rendering the problem  
42 unsolvable. A3 means that a more preferred item is more likely to win a comparison. If it is not true, the meaning of  
43 “more preferred” becomes vague and we may not have enough information to recover the ranking. **7) Minor.** Thanks.  
44 We will make the introduction of the shorthand MNL more properly in the final version.

45 **Reviewer #3. Question 1. p1, p2)** (i) Theorem 2 is not a corollary of Theorem 3. Theorem 2 states an instance-wise  
46 lower bound for all  $\delta$ -correct algorithms and input instances satisfying A1-A3, while Theorem 3 only works for the  
47 MNL model. (ii) In Theorem 3, the algorithms know *a priori* that the input instances satisfy the MNL model, but  
48 in Theorem 2, the algorithms do not. Thus, Theorem 3 is also not a corollary of Theorem 2. **p3)** (i) Eqn(1) is no  
49 larger than Eqn(2). The right hand side of Eqn(1) (i.e.,  $\min\{\dots\}$ ) is no larger than  $\sum_{i \in [n]} \tilde{\Delta}_i^{-2} \log n$  (by letting each  
50  $x_i = 1/n$ ). Summing up the left hand side of Eqn(1) (i.e.,  $\sum_{i \in [n]} [\dots]$ ) and  $\sum_{i \in [n]} \tilde{\Delta}_i^{-2} \log n$ , we get Eqn(2). Thus,  
51 Eqn(1)  $\preceq$  Eqn(2). (ii) There are cases where Eqn(1) is strictly lower than Eqn(2). For example, if  $\tilde{\Delta}_1 = n^{-2}$  and  
52  $\tilde{\Delta}_i = 0.1$  for all  $i \geq 2$ , then the right hand side of Eqn(1) is  $\Omega(\tilde{\Delta}_1^{-2} + \sum_{i \geq 2} \tilde{\Delta}_i^{-2} \log n)$  (by letting  $x_1 = 1/2$ ), lower  
53 than  $\sum_{i \in [n]} \tilde{\Delta}_i^{-2} \log n$ , making Eqn(1)  $\prec$  Eqn(2). **Question 2.** (i) PLPAC-AMPR only works for the MNL model, so  
54 it only occurs in Figure 2(b) type-MNL. (ii) Active Ranking is for finding the Borda ranking [15]. For type-random  
55 instances, Borda ranking may not be the same as the exact ranking, so we do not compare Active Ranking in this case  
56 (so not present it in Figure 2(c) type-random). **Typos. p1)** Yes, we agree, and we will add the clarification in the final  
57 version. **p2)** The “2” near  $O(\dots)$  in Line 87 is a footnote index, not “square”. We will make it clearer in the final version.