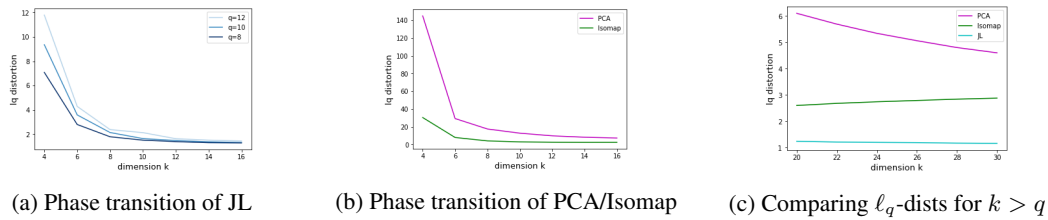
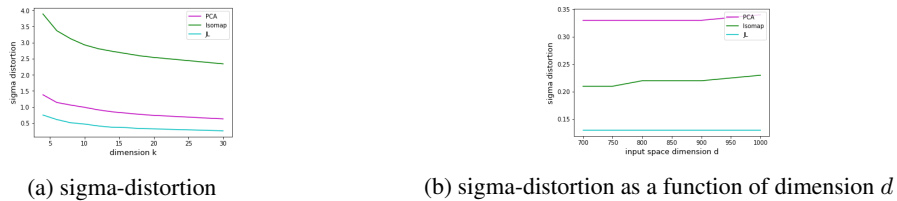


1 We first address all referees’ request to include experimental validation of the theoretical results in the paper.
2 **Empirical Experiments.** In all the experiments, we have followed VL [NIPS’18] using similar distributions for
3 sampling random Euclidean input spaces, tests were made for a large range of parameters, averaging over at least 10
4 independent tests. The results are consistent for all settings, and measures, and will be provided in full in the final paper.
5 *Tightness of the bounds, phase transition phenomenon, and superiority of JL.* In our paper we proved theoretical bounds
6 for the distortion measures of the JL transform into $k \geq 1$ dimensions. In particular, we showed that for $q < k$ the
7 ℓ_q -distortion is bounded by $1 + O(1/\sqrt{k}) + O(q/k)$, and all the rest measures are bounded by $O(\sqrt{q/k})$. Particularly,
8 the bounds are independent of n - the size and dimension d of the input data set. In addition, we proved that for the
9 ℓ_q -distortion and REM_q measures a phase transition must occur at $q \sim k$ for any dim. reduction method, where the
10 bounds dramatically increase from being bounded by some constant to grow with n , in particular as $\text{poly}(n)$ for $q > k$.
11 The graphs in Fig. 1 and Fig.2a describe the following setting: A random X of a fixed size and dimension $n = 800$ was
12 embedded into $k \in [4, 30]$ dimensions, by the JL/PCA/Isomap methods; the value of $q = 10$. We stress that we run
13 many more experiments a wide range of parameter values of $n \in [100, 3000]$, $k \in [2, 100]$, $q \in [1, 10]$, and obtained
14 essentially identical qualitative behavior. In Fig. 1a, the ℓ_q -distortion as a function of k of the JL embedding is shown
15 for $q = 8, 10, 12$. The phase transitions are seen at around $k \sim q$ as predicted by our theorems. In Fig. 1b the bounds
16 and the phase transitions of the PCA and Isomap methods are presented for the same setting ($d = 800, q = 10$), as
17 predicted by our lower bounds. In Fig. 1c, ℓ_q -distortion bounds are shown for increasing values of $k > q$. Note that
18 the ℓ_q -distortion of the JL is a *small constant close to 1*, as predicted, compared to values significantly > 2 for the
19 compared heuristics. Overall, Fig. 1 clearly shows that JL dramatically outperforms the other methods for all the
range of values of k . Below is **Fig. 1: Validating ℓ_q -distortion behavior.** The same conclusions as above hold for



20 σ -distortion as well, as shown in Fig. 2a, on the same sample data set. In the last experiment shown in Fig. 2b, we tested
21 the behavior of the σ -distortion as a function of d -the dimension of the input data set, similarly to that of VL[18](Fig.
22 2), and tests are shown for embedding dimension $k = 20$ and $q = 2$. According to our theorems, the σ -dist of the JL
23 transform is $O(\sqrt{q/k})$, which is bounded by constant for $q < k$. It is seen that the σ -dist is growing as d increases for
24 both PCA/ISOMAP, whereas it is a *constant* for JL, as predicted. Moreover, JL obtains a *significantly smaller value* of
25 σ -distortion. Below is **Fig. 2: Validating σ -distortion behavior.** In the final paper, we will include further experiments



26 on the JL-based approximation algorithms, which are expected to show similar to more dramatic qualitative behavior.
27

28 **Discussion/Conclusion Section.** Two of the referees #2 and #5 rightfully requested the inclusion of such a section,
29 discussing consequences of the work for practical considerations. We note that a shortened version implicitly appears in
30 the last 3 paragraphs prior to section 1.1 in the supp. material. The discussion section will greatly expand on these.

31 **Further improvement suggestions.** We thank referee #2 for his detailed comments that we’ll happily incorporate.
32 We shall adopt referee #4’s suggestion to use numeric citations (we didn’t realize it was possible). Referee #5 asks to
33 improve clarity and writing, in contrast to the others who seem impressed by it. He mentioned “sections that do not exist
34 (on page 3)” - can be found in supp. material. We realize that NIPS has a wide range of audience and we will make an
35 effort to rewrite in a way that will be clear for all. The referee also criticizes the theoretical methodology of the paper,
36 yet the paper contains very detailed proofs for all theorems. The *only* exception mentioned by the referee: proofs of the
37 properties in page 5 will be included in the full paper. In particular, the “translation invariance” property mentioned by
38 the referee, trivially holds for any distance based measure in any metric space *by definition*. We note that we do not
39 “propose a new distortion measure” but new dim. reduction methods, based on JL, which we have addressed above.