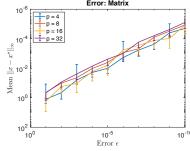
- We thank all the reviewers for their helpful comments and suggestions. Below we address the concerns raised.
- **Importance of Convergence vs Function value (R1).** For an algorithm with a $\log \frac{1}{\varepsilon}$ dependence of the running time 2
- for computing a $(1+\varepsilon)$ -approximate solution, like p-IRLS, the guarantee can be translated into a guarantee for 3
- convergence in the solution without any significant loss in the runtime complexity of the method. We demonstrate this 4
- theoretically and experimentally below. We thank the reviewer for pointing out that this is inadequately explained in the
- paper, and we will clarify this in the final version of the paper.
- If x is a $(1 + \delta)$ -approximate solution, using Lemma A.1 from the supplementary material we can show that
- we can achieve the guarantee $\|\boldsymbol{x}-\boldsymbol{x}^\star\|_{\infty} \leq \varepsilon \|\boldsymbol{A}\boldsymbol{x}^\star-\boldsymbol{b}\|_p$ by picking $\delta = \left(\frac{\varepsilon\sigma_{\min}(\boldsymbol{A})}{4m}\right)^p$, where $\sigma_{\min}(\boldsymbol{A})$ is the smallest singular value of \boldsymbol{A} . This gives $\log \frac{m}{\delta} = O(p\log \frac{m}{\sigma_{\min}(\boldsymbol{A})\varepsilon})$, and hence a total iteration count of
- $O(p^{4.5}m^{\frac{p-2}{2(p-1)}}\log\frac{m}{\sigma_{\min}(\mathbf{A})\varepsilon})$. Asymptotically, the running time bound is only off by a factor of p if we wish to 10
 - measure the convergence in ℓ_{∞} -norm, as long as $\log \frac{1}{\sigma_{\min}(A)} = O(\log \frac{m}{\varepsilon})$.



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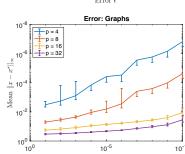
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We also demonstrate this relation experimentally. The plots demonstrate the average resulting ℓ_{∞} norm deviation for the solution computed, as we change the ε parameter used in the algorithm. We use the instances described in the paper; matrices of size 1000×800 and graphs with 1000 nodes. For each instance, we: 1) find a very high accuracy solution, by choosing a very small $\varepsilon \sim 10^{-25}$, 2) scale the problem so that the optimum value is 1, and run the algorithm again to find the optimum solution x^* . 3) Now we have a problem such that $||Ax^* - b||_p = 1$, we run the algorithm again with various values of arepsilon, to obtain solutions $m{x}(arepsilon)$ and plot $\|x(\varepsilon) - x^*\|_{\infty}$ (averaged over 20 samples). These results are very much in agreement with the theoretical $\varepsilon^{\frac{1}{p}}$ dependence proved above. (Note that the error bars indicate $\log(\text{mean} \pm \text{std})$ so they are missing on one side when mean < std.)



Runtime comparison with [AKPS19] and [BCLL18] (R1). As noted by R1, the running time of [AKPS19] (and [BCLL18]) is not stated precisely in the comparison. The running time bounds are not stated precisely in either paper; they hide the p dependencies and poly $(\log \frac{m}{\varepsilon})$ dependencies. We have focused on the polynomial terms in the comparison because they are the dominant terms. For [AKPS19] the running time is at least $p^{2p+2}m^{\frac{p-2}{3p-2}}\log^2\frac{m}{\varepsilon}$, for [BCLL18] it seems to be at least $p^{2.5}m^{\frac{p-2}{2p}}\log^2\frac{m}{\varepsilon}$. The $\log^2\frac{m}{\varepsilon}$ dependence is worse for both [AKPS19] and [BCLL18], compared to our algorithm, and the p^{2p+2} factor is much worse in [AKPS19]. We will clarify this in the paper.

Experimental comparison to [AKPS19] and [BCLL18] (R1). We agree that a direct comparison to [AKPS19] and [BCLL18] is desirable. Unfortunately, both algorithms are quite complicated to implement, and no implementations are publicly available. The [BCLL18] paper lacks an explicit algorithm description and leaves out several details (e.g. it asks to run accelerated gradient descent (AGD) "until convergence", the specific accuracy target for AGD will have a large impact on the running time). The [AKPS19] algorithm description also leaves out specifying several parameters in the algorithm, hiding p dependencies and $\log \frac{m}{c}$ factors. As pointed out above, these large hidden factors make the algorithm, as stated, difficult to implement efficiently. In contrast, our algorithm is far simpler to implement.

- Simplicity of p-IRLS compared to [MPT+18] (R3). We thank R3 for this. We will clarify this in the final version. 39
- Combining p-norm with a regularizer e.g. ℓ_1 (Lasso) (R3). This is definitely a great idea for future work. Our 40 current techniques would not suffice for this, but we thank the reviewer for pointing out this potential direction. 41
- Spacing between subfigures in figure 4 (R1) We will address this in the final version.
- **Proof of claimed bound.** We prove the bound on $\|x x^*\|_{\infty}$ claimed above. Given that x is a $(1 + \delta)$ -approximate solution, using Lemma A.1, we can write the following lower bound on the objective value: 43

$$(1+\delta) \|\boldsymbol{A}\boldsymbol{x}^{\star} - \boldsymbol{b}\|_{p}^{p} \geq \|\boldsymbol{A}\boldsymbol{x}^{\star} - \boldsymbol{b}\|_{p}^{p} + p \left(\boldsymbol{A}\boldsymbol{x}^{\star} - \boldsymbol{b}\right)^{\top} \boldsymbol{R}\boldsymbol{A}(\boldsymbol{x} - \boldsymbol{x}^{\star}) + \frac{p}{8}\boldsymbol{A}(\boldsymbol{x} - \boldsymbol{x}^{\star})^{\top} \boldsymbol{A}^{\top} \boldsymbol{R}\boldsymbol{A}(\boldsymbol{x} - \boldsymbol{x})^{\star} + 2^{-(p+1)} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{A}\boldsymbol{x}^{\star}\|_{p}^{p}$$

- where $R = \text{diag}(|Ax^{\star} b|^{p-2})$. Since the gradient at x^{\star} is 0, simplifying, we get, $2^{p+1}\delta \|Ax^{\star} b\|_p^p \ge 1$
- $\|Ax Ax^{\star}\|_{n}^{p}$. Now, translating between various norms, we obtain,

$$\|oldsymbol{x} - oldsymbol{x}^\star\|_{\infty} \leq rac{1}{\sigma_{\min}(oldsymbol{A})} \|oldsymbol{A}oldsymbol{x} - oldsymbol{A}oldsymbol{x}^\star\|_2 \leq rac{m^{rac{1}{2} - rac{1}{p}}}{\sigma_{\min}(oldsymbol{A})} \|oldsymbol{A}oldsymbol{x} - oldsymbol{A}oldsymbol{x}^\star\|_p \leq rac{2m^{rac{1}{2}}}{\sigma_{\min}(oldsymbol{A})} \left(rac{2\delta}{m}
ight)^{rac{1}{p}} \|oldsymbol{A}oldsymbol{x}^\star - oldsymbol{b}\|_p \,.$$